BASICS OF SEMICONDUCTORS

From Andras Gyenis
University of Colorado Boulder

GOAL: MAKE A QUBIT FROM THE STATES OF ELECTRONS

FOR EXAMPLE: $|\uparrow\rangle$, $|\downarrow\rangle$

⇒ WE NEED TO CONFINE SINGLE ELECTRONS

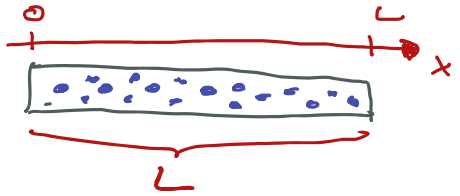
→ USE SEMICONDUCTORS TO DO THAT.

WHAT ARE SEMICONDUCTORS? → BAND THEORY

- RESISTIVITY FALLS BETWEEN METAL & INSULATORS
- $R \propto e^{E_0/k_B T}$

1D FREE ELECTRONS IN A BOX.

N ELECTRONS



$$H = \sum_i \frac{p_i^2}{2m} \quad \text{(NO POTENTIAL ENERGY)}$$

$$p_i = -i\hbar\partial_x^2$$

BOUNDARY CONDITION: $\Psi(x=0) = 0$.

$\Psi(x=L) = 0$.

$$\Psi_n(x) = \sqrt{\frac{2}{L}} \cdot \sin\left(n \cdot \frac{\pi}{L} \cdot x\right)$$

k WAVE VECTOR OF THE ELECTRON $\qquad \hbar k = p$

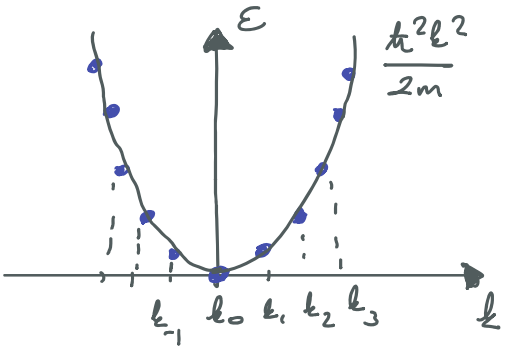
IT IS BETTER TO USE PERIODIC $\Psi(x=0) = \Psi(x=L)$

$$H\psi_k(x) = \frac{\hbar^2}{2m} \cdot k^2 \psi_k(x)$$

$$\psi(x) = e^{ikx}$$

$$k = \frac{2\pi}{L} \cdot n$$

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m} \rightarrow \text{ENERGY - MOMENTUM DISPERSION}$$

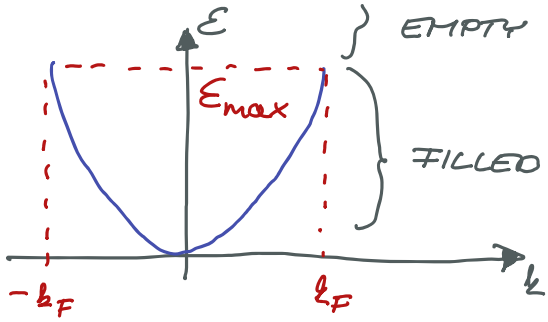


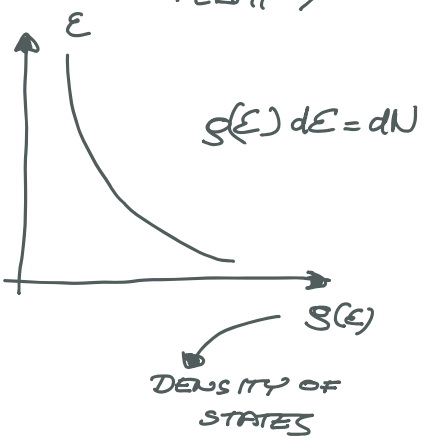
PAULI PRINCIPLE:
ONE STATE CAN BE OCCUPIED BY TWO ELECTRON (↑, ↓ SPINS)

$$\Delta k = \frac{\pi}{L} \quad \text{IF } L \to \infty, \text{ CONTINUOUS SPECTRUM}$$

MOMENTUM WITH THE HIGHEST ENERGY:
FERMI MOMENTUM: k_F WITH ENERGY $\varepsilon_{FERMI} = \mu$

CHEMICAL POTENTIAL
COST OF ADDING 1 ELECTRON





$$2k_F = \Delta k \cdot \frac{N}{2}$$

$\Delta k = \frac{\pi}{L}$ ↳ DOUBLE OCCUPANCY

$$k_F = \frac{\pi}{4} \cdot n$$

↳ DENSITY

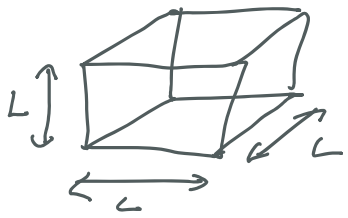
$$\varepsilon_k = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{\pi}{4L} N\right)^2 \rightarrow N \propto \varepsilon^{1/2}$$

$$g(\varepsilon) = \frac{dN}{d\varepsilon} \propto \varepsilon^{-1/2}$$

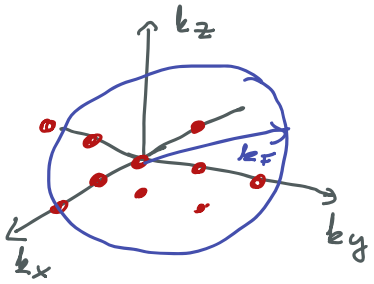
CALCULATING THE DENSITY OF STATES

FOR EXAMPLE IN 3D:

$$H = \sum_i \frac{p_{xi}^2}{2m} + \frac{p_{yi}^2}{2m} + \frac{p_{zi}^2}{2m}$$



$$\psi_{\vec{k}}(\vec{r}) = A \sin(k_x x) \sin(k_y y) \sin(k_z z)$$



$$\Delta k_x = \Delta k_y = \Delta k_z = \frac{2\pi}{L}$$

NUMBER OF STATES: $N = 2 \cdot \frac{\frac{4\pi}{3} k_F^3}{(2\pi/L)^3} = \frac{V}{3\pi^2} k_F^3$

SPIN $\qquad L^3$

$$\rightarrow \quad \varepsilon_F = \frac{\hbar^2}{2m} k_F^2 = \frac{\hbar^2}{2m} \left(\frac{3\pi^2 N}{V} \right)^{2/3}$$

$$N = \frac{V}{3\pi^2} \left(\frac{2m\varepsilon}{\hbar} \right)^{3/2}$$

$\rightarrow$ DENSITY OF STATES: $g(\varepsilon) d\varepsilon = dN$

$$g(\varepsilon) = \frac{dN}{d\varepsilon} \propto \varepsilon^{1/2}.$$

IN 2D: $g(\varepsilon)$ IS CONSTANT

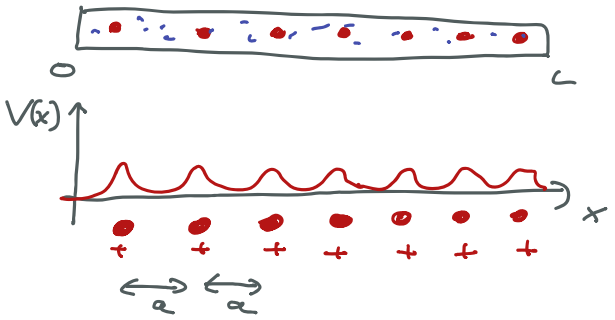
IN 1D: $g(\varepsilon) \propto \varepsilon^{-1/2}$

WHY?

$I \tilde{=} e g \cdot v$

BUT THERE ARE ATOMS → PERIODIC POTENTIAL

ELECTRONS IN 1D PERIODIC LATTICE



$$H = \sum_i \frac{p_i^2}{2m} + V(r) \qquad V(r) = V(r+a)$$

BLOCH - THEORY :

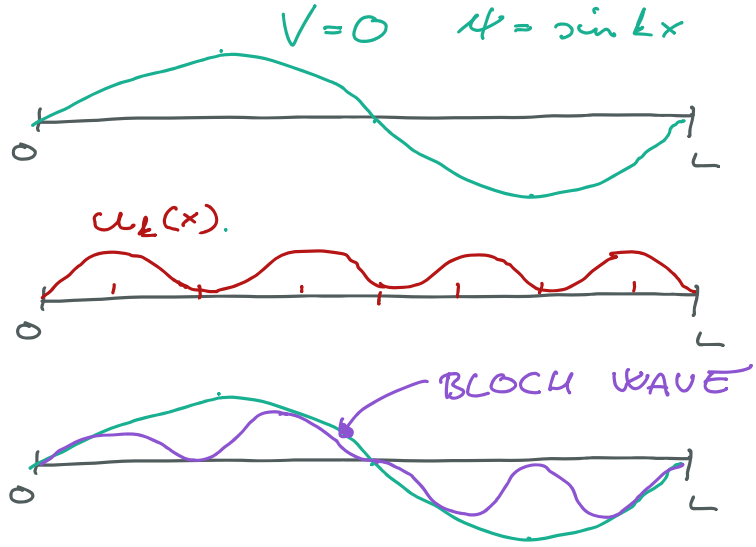
$u(r) = u(r+a)$

LATTICE PERIODIC FUNCTION

CORRECTION TERM

$$\psi_k(x) = e^{ikx} \cdot u_k(x)$$

FOR EXAMPLE



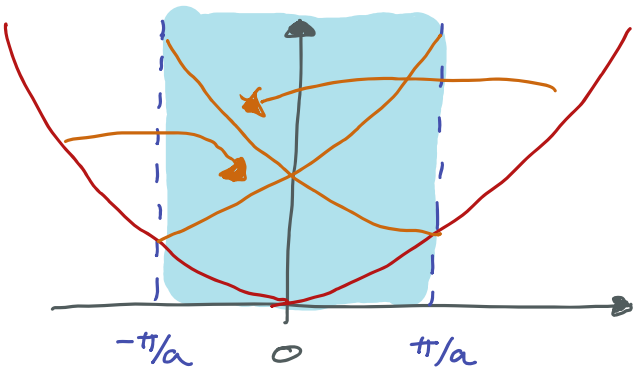
INTRODUCE RECIPROCAL WAVEVECTORS:

$$G_j = \frac{2\pi}{a} \cdot j$$

IT CAN BE SHOWN THAT k AND $k' = k + G_j$ ARE EQUIVALENT.

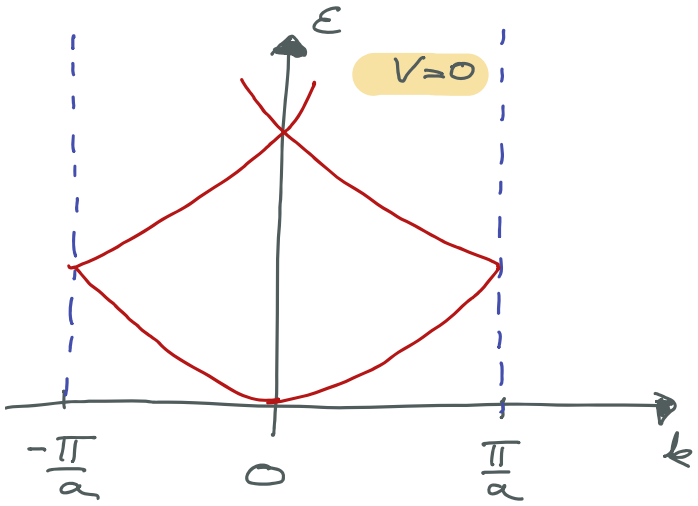
$$\psi_k(x) = \psi_{k+G}(x)$$

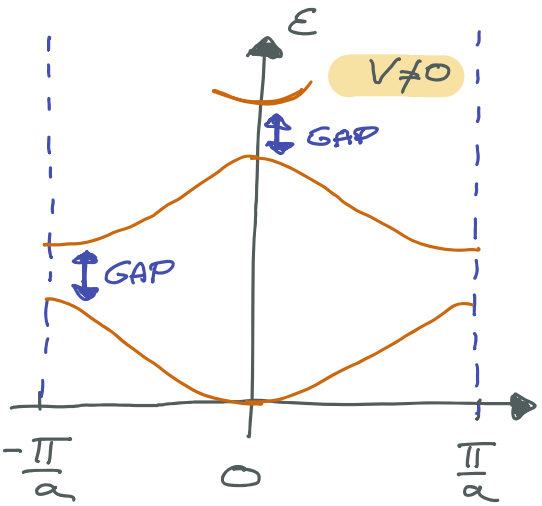
$$\mathcal{E}_k = \mathcal{E}_{k+G}$$



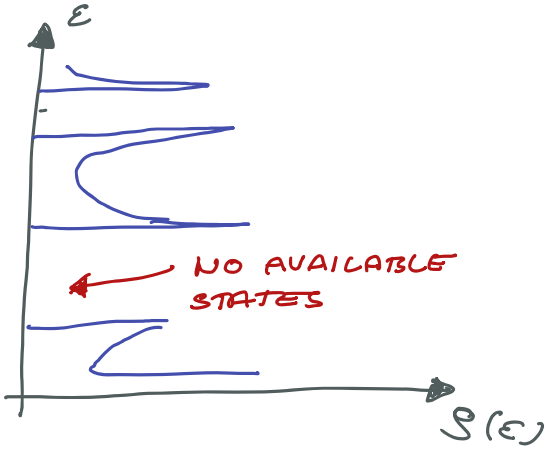
WE CAN DESCRIBE THE SYSTEM IN THE FIRST BRILLOUIN ZONE

IF $V \neq 0$ BUT SMALL, THE DEGENERACY IS LIFTED:

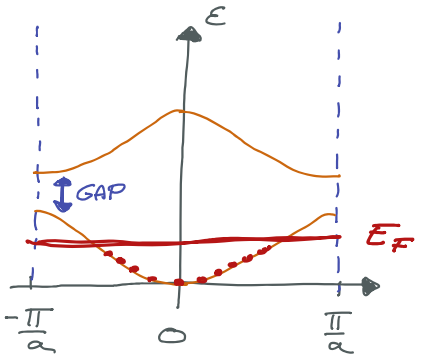




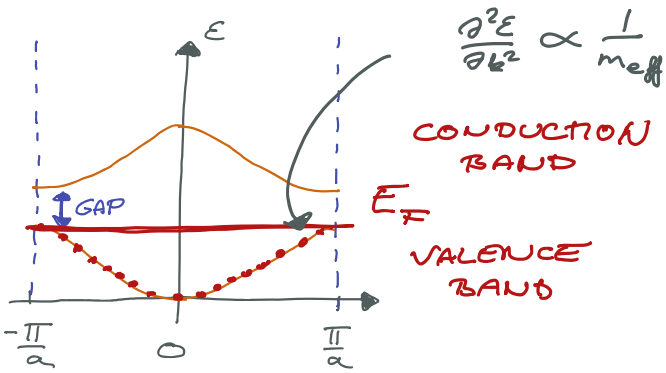
DENSITY OF STATES



METALS

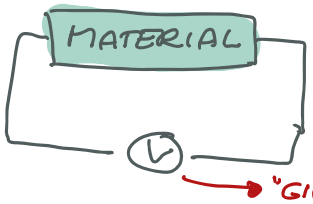


INSULATORS OR SEMICONDUCTOR



$$N = 2\frac{2\pi/a}{2\pi/L} = \frac{2L}{a}$$ → 2 ELECTRON / SITE

INSULATOR



I = ?

→ "GIVES ENERGY" TO THE ELECTRON

ELEMENTARY SEMICONDUCTORS

MOSTLY LOCATED IN GROUP IV:

C (~5eV), Si (1.1eV), Ge (0.7eV)
(INSULATOR)

THESE ARE INDIRECT-GAP SEMICONDUCTORS $k_V = 0$ MIN, $k_C \neq 0$ MAX

DIAMOND LATTICE STRUCTURE

COMPOUND SEMICONDUCTORS:

- BINARY COMPOUNDS

GROUP IV: SiC (2.4eV)

GROUP III-V: InAs (0.36eV), InP (1.3eV), GaAs (1.4eV)

II-VI: CdTe (1.4eV)

GaAs & InSb DIRECT-GAP SEMICONDUCTORS

ZINC BLENDE STRUCTURE, IDENTICAL TO DIAMOND BUT DIFFERENT NEIGHBORS

- TERNARY COMPOUNDS

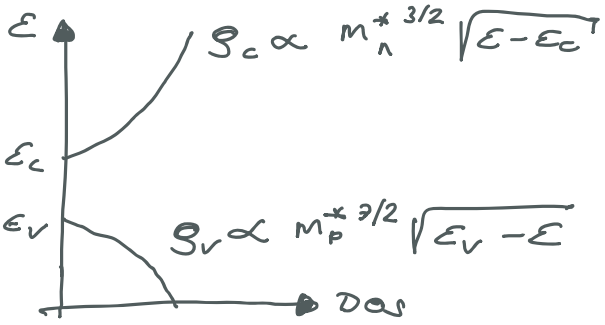
$Al_x Ga_{1-x} As$

$In_x Ga_{1-x} As$ → BANDGAP ENGINEERING

INTRINSIC SEMICONDUCTORS:

(THE ABOVE EXAMPLES)

CHARGE CARRIERS CAN BE GENERATED BY THERMAL EXCITATIONS



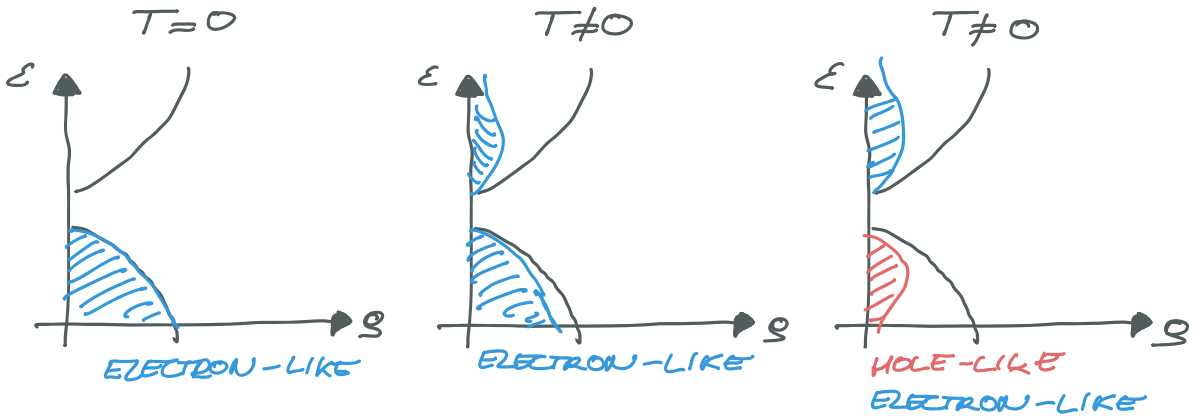
THERMAL OCCUPATION IS:

$$n_c(T) = \int_{\mathcal{E}_c}^{\infty} g_c(\mathcal{E})\, f(\mathcal{E})\, d\mathcal{E}$$

$$p_v(T) = \int_{-\infty}^{\mathcal{E}_v} g_v(\mathcal{E})\, \tilde{f}(\mathcal{E})\, d\mathcal{E}$$

$$f(\mathcal{E}) = \left[1 + e^{(\mathcal{E}-\mu)/T}\right]^{-1}$$

$$\tilde{f}(\mathcal{E}) = \left[1 + e^{(\mu-\mathcal{E})/T}\right]^{-1}$$



$n(T) = p(T)$ INTRINSIC CARRIER DENSITY.

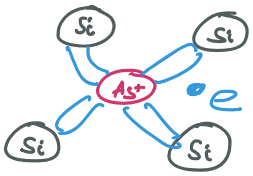
FOR EXAMPLE, $n^{Si}(300K) = 10^{10}/cm^3$

$n^{Ge}(300K) = 10^{13}/cm^3$

FROM $n(T) = p(T) \Rightarrow \mu \simeq \frac{\mathcal{E}_c + \mathcal{E}_v}{2} + \underbrace{k_B T \frac{3}{4} \ln \frac{m_p^*}{m_n^*}}_{\text{SMALL}}$

DOPED SEMICONDUCTORS "SHALLOW DONORS"

(EXTRINSIC SEMICONDUCTORS)

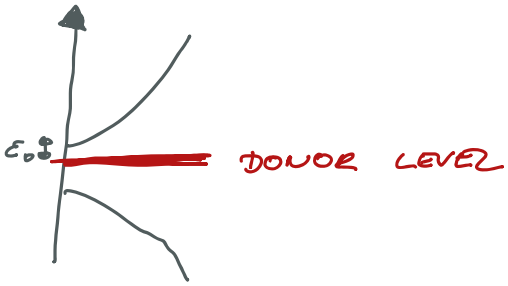


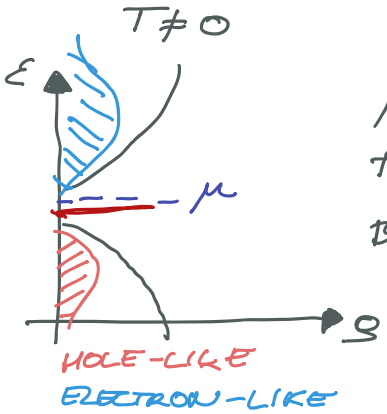
DONORS : BOUND STATE APREAR AROUND THE IMPURITY

$\mathcal{E}_r + m^* \rightarrow \mathcal{E}_0 \approx 10meV$

$r_0 \approx 50\text{Å}$

BOHR RADIUS

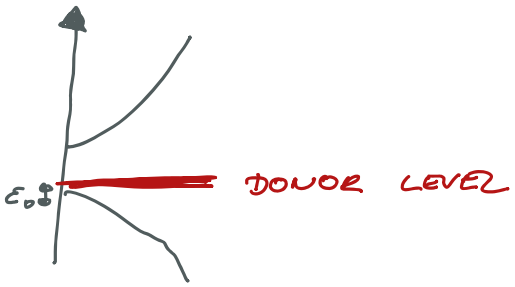




μ CHEMICAL POTENTIAL MOVES BETWEEN THE DONOR LEVEL AND CONDUCTION BANDS

N-TYPE SEMICONDUCTORS

CARRIERS ARE CONDUCTION ELECTRONS FROM DONOR LEVELS



P-TYPE

HOLES COMING FROM ACCEPTORS

→ A POSSIBLE WAY TO CONFINE ELECTRONS A QUBIT.

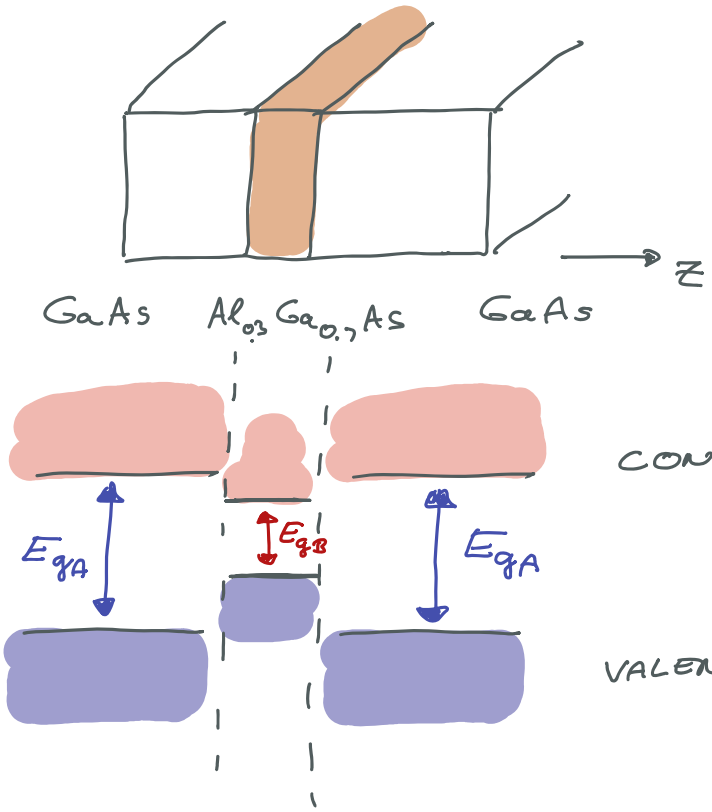
BAND ENGINEERING

CREATE POTENTIALS BETWEEN MATERIAL INTERFACES

MATERIALS WITH THE SAME LATTICE CONSTANT AND CRYSTAL STRUCTURE CAN BE GROWN ON TOP OF EACH OTHER.

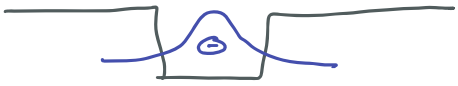
DIFFERENT BAND GAPS BUT SAME LATTICE CONSTANT

THE SIMPLEST EXAMPLE FOR TWO-DIMENSIONAL ELECTRON GAS:



CONDUCTION BAND

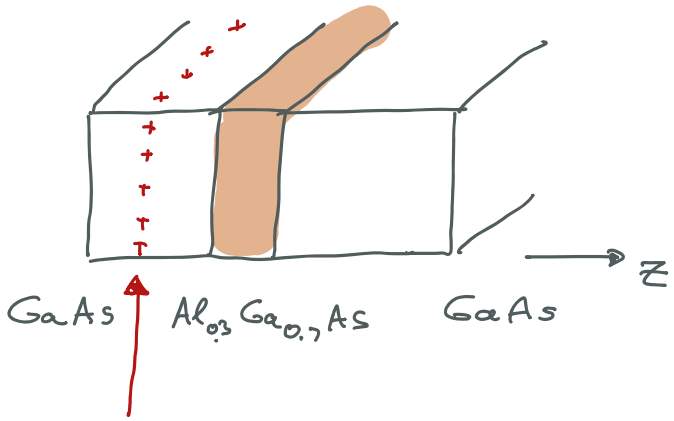
VALENCE BAND



QUANTUM WELL CONFINED IN 1 DIMENSION

BUT WE STILL NEED TO ADD CARRIERS:

δ - DOPING (DONORS AWAY FROM THE WELL)



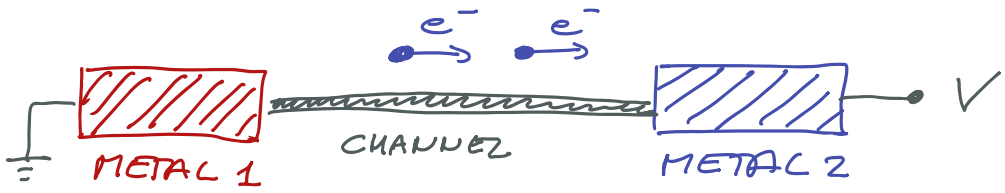
δ-DOPED PLANE : SCATTERING CENTERS ARE FAR FROM THE 2DEG

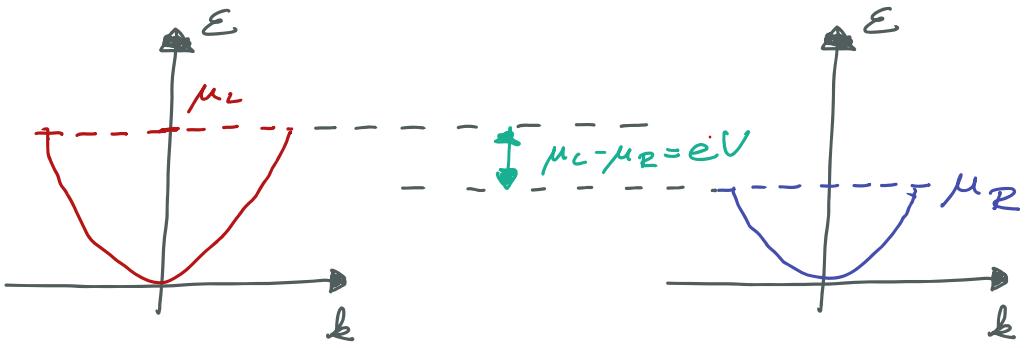
WHAT ABOUT X/Y MOVEMENT?

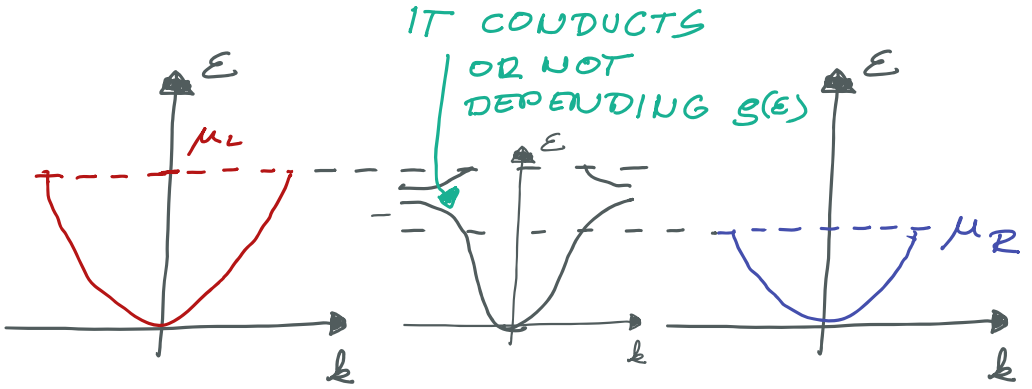
- MAKE IT SMALL! $\lambda_F \sim$ DIMENSION
- ADD GATES

CHEMICAL POTENTIAL

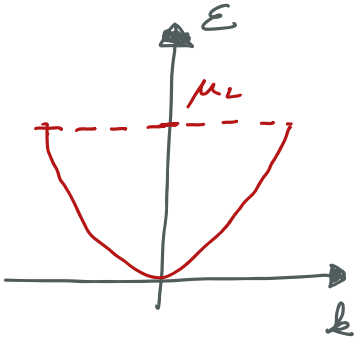
THE ENERGY COST OF ADDING ONE ELECTRON TO THE SYSTEM ($\approx E_F$)

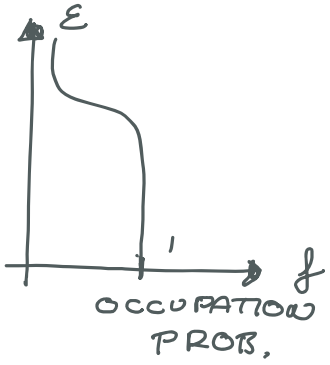






AT FINITE TEMPERATURES





$$f(E-\mu) = \frac{1}{1+e^{(E-\mu)/k_B T}}$$

QUANTUM TRANSPORT

CURRENT: $I = \frac{\Delta Q}{\Delta t} = \frac{N \cdot e \cdot v \cdot \Delta t}{\Delta t} =$

$$= N \cdot e \cdot v$$

IN A QUANTUM VERSION

$N \longrightarrow g(E) f(E)$

$v \longrightarrow \hbar k/m$

CURRENT TO THE RIGHT:

$$\int dE \; f(E-\mu_R) \; g(E) e \frac{\hbar k(E)}{m}$$

CURRENT TO THE LEFT

$$\int dE \; f(E-\mu_L) \; g(E) e \frac{\hbar k(E)}{m}$$

$$I_{TOTAL} \propto \int dE \underbrace{\left[f(E-\mu_R) - f(E-\mu_L) \right]}_{\cdot eV} g(E) \frac{\hbar k(E)}{m}$$

$$I_{TOTAL} = G \cdot V$$

↳ CONDUCTANCE

FOR PERFECT TRANSITON IN 1D

$$G = \frac{2e^2}{h}$$

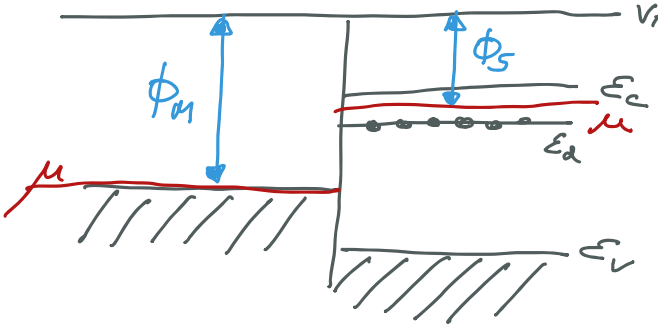
METAL - SEMICONDUCTOR INTERFACE

- ANDERSON'S RULE : THE VACUUM LEVELS SHOULD HAVE THE SAME ENERGY

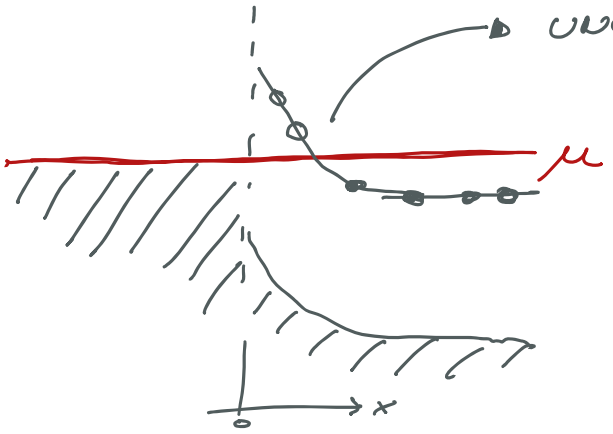
- WORK FUNCTION: ENERGY INVESTED TO REMOVE AN ELECTRON $X \rightarrow X^+ + e^-$

- ELECTRON AFFINITY: ENERGY RELEASED WHEN ELECTRON IS ATTACHED TO THE BOTTOM OF THE CONDUCTION BAND

METAL : SEMICOND → N-DOPED GaAs



ELECTRONS GO FROM SM TO METAL TO EQUALIZE THE CHEMICAL POTENTIALS

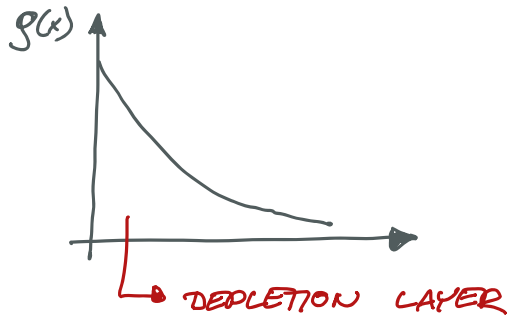


UNCOMPENSATED IONS NONUNIFORM ELECTRICAL FIELD

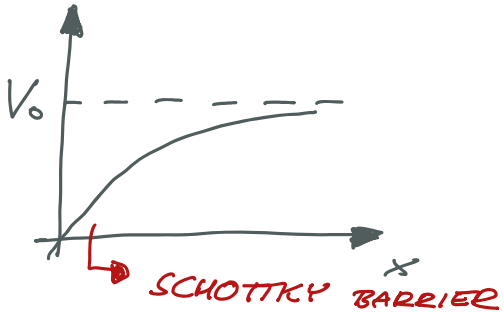
$$\Delta E = -eV(r)$$

CONTACT POTENTIAL DIFFERENCE: $eV_0 = \phi_M - \phi_S$

CHARGE DISTRIBUTION:

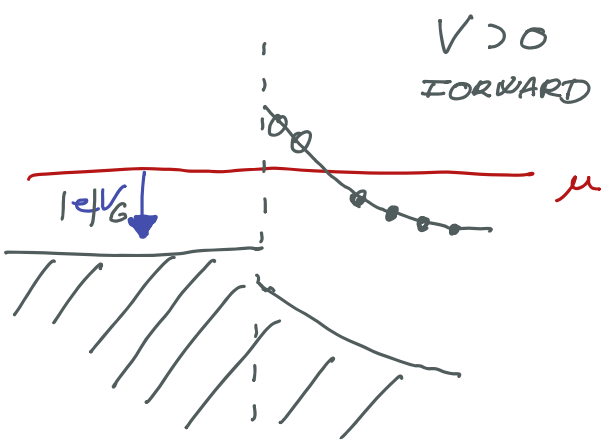


INDUCED POTENTIAL:



FIELD EFFECT:

APPLYING VOLTAGE TO THE GATE:

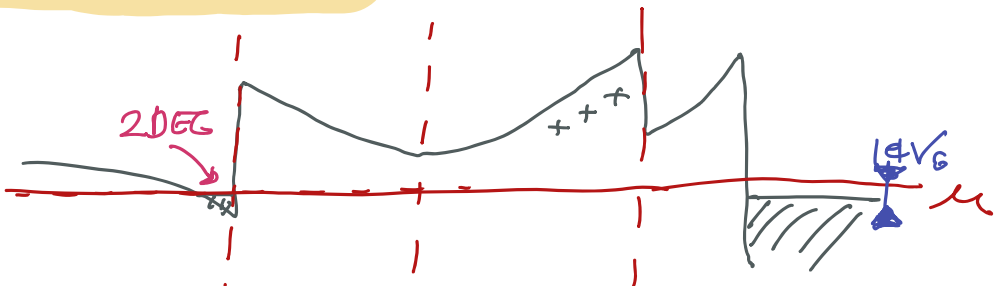


ALSO: FIELD-EFFECT

AT $V_G = 0$, in GaAs

$d \approx 60$ nm

HETEROSTRUCTURE



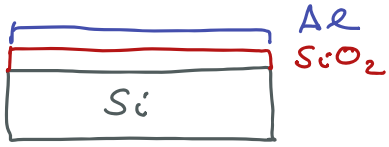
"PARALLEL PLATE CAPACITOR" MODEL

$$C = \varepsilon_r \frac{A}{d}$$

$$Q = C V_g \longrightarrow \delta n_s = -\varepsilon_r \frac{1}{d} V_G .$$

MOSFET (TRANSISTOR)

↳ METAL OXIDE SEMICONDUCTOR FIELD EFFECT TRANSISTOR



PART 6 :

QUANTUM DOTS

- HOW SMALL CAN WE MAKE TRANSISTORS WITHOUT CHANGING THE WAY THEY WORK?

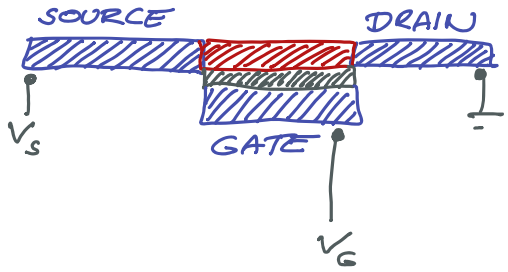
OR:

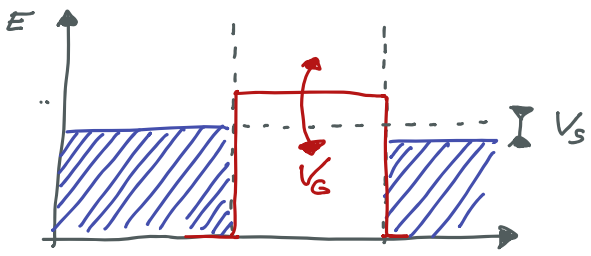
- HOW SMALL DO WE HAVE TO MAKE THEM TO GET NEW FEATURE?

SINGLE ELECTRON TRANSISTOR

- THE FIRST "ARTIFICIAL ATOM" TO REALIZE A QUBIT.
- "QUANTUM AMPLIFIER" → READ OUT THE STATES OF QUBITS.

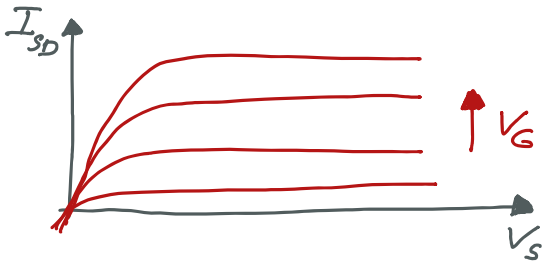
1. CONVENTIONAL TRANSISTOR



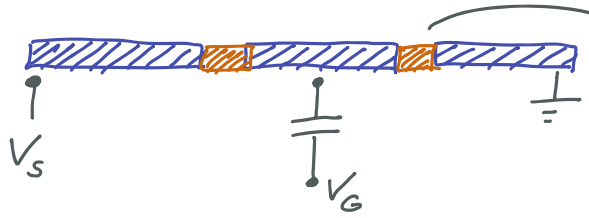


$V_G = 0$ NO CONDUCTION ELECTRONS IN THE CHANNEL

$V_G > V_{TH}$: CURRENT FLOWS.



2. OPERATING PRINCIPLE OF SET



IF TUNNEL BARRIER IS LARGE ENOUGH, THE NUMBER OF ELECTRONS QUANTIZED $Q = Ne$.

$$E_C = \frac{e^2}{2C}$$

(1.) CHARGING ENERGY EXCEEDS THE TEMPERATURE FLUCTUATIONS.

$$E_C \gg k_B T$$

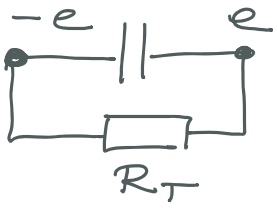
CAPACITANCE OF A FLAT DISK:

$$C = 8\varepsilon_R \varepsilon_0 R$$

$$\rightarrow \frac{e^2}{16\varepsilon_R \varepsilon_0 R} >> k_B T$$

MAKE THE ISLAND SMALL.

② TUNNELING OUT FROM THE ISLAND SHOULD BE SMALL:



$$\Delta\tau = R_T C$$

HEISENBERG UNCERTAINTY RELATION:

$$\Delta E \Delta\tau > h$$

$$E_c >> \Delta E = h/\Delta\tau = h/R_T C$$

$$\Updownarrow$$

$$R_T >> 2\frac{h}{e^2}$$

RESISTANCE QUANTUM:

$R_Q = 25\,k\Omega$

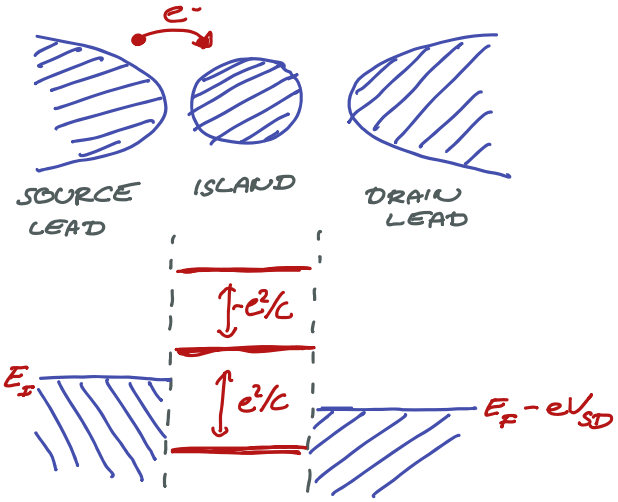
- AS THE SIZE OF THE ATOM INCREASES:
  - ↘ COULOMB ENERGY
  - ↘↘ ORBITAL ENERGY

- ELECTRONIC DEVICES CANNOT BE SEEN AS SEAS OF THOUSANDS OF ELECTRONS

- CONVENTIONAL TRANSITOR TURNS ON WHEN AN ELECTRON ADDED TO IT, SET TURNS ON AND OFF EVERY TIME AN ELECTRON IS ADDED.

- FIRST OBSERVATION WAS IN NARROW TRANSISTORS AT LOW TEMPERATURES.

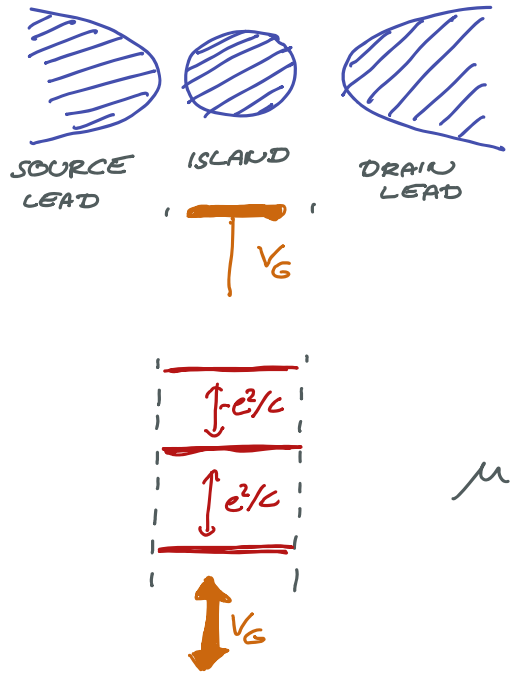
COULOMB BLOCKADE



ADDING AN ELECTRON TO THE ISLAND COSTS $-e^2/2C$ ENERGY

→ ENERGY GAP

FOR THE CURRENT TO FLOW: (1) ELECTRON IN THE SOURCE.
(2) HOLE IN THE DRAIN.



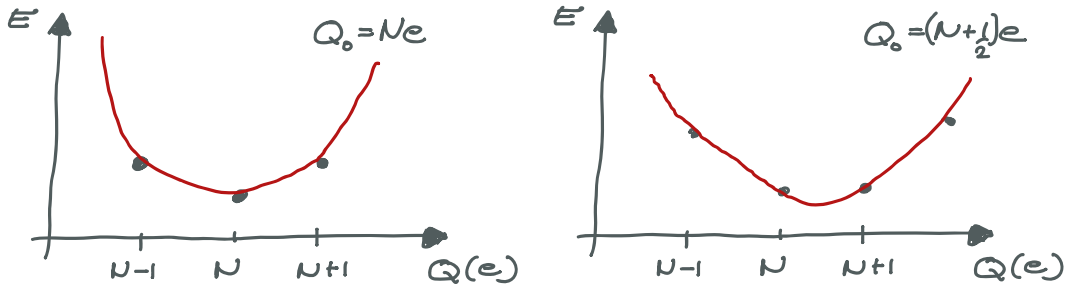
ELECTROSTATIC ENERGY OF THE CHARGE Q ON THE ISLAND:

$$E = -QV_G + Q^2/2C =$$
$$= -Ne \cdot V_G + (Ne)^2/2C$$
$$= (Q-Q_0)^2/2C + \text{const.}$$
$$Q_0 = CV_G$$

$$\mu = E(N+1) - E(N) =$$
$$= -eV_G + [(N+1)^2 - N^2]\frac{e^2}{2C} =$$
$$= -eV_G + N\frac{e^2}{C} + \text{const.}$$

WHEN CAN A CURRENT FLOW THROUGH THE DOT?

$$E_N = E_{N+1}$$



BUT! COULOMB ENERGY IS ONLY HALF OF THE STORY

→ PAULI EXCLUSION PRINCIPLE REQUIRES ELECTRONS

TO BE IN DIFFERENT QUANTUM LEVELS.

FUN FACT: IN REAL ATOMS ELECTRON-ELECTRON INTERACTION IS LESS IMPORTANT.

CONDITION FOR ELECTRON TO HOP ON THE DOT:

$$\mathcal{F}(N+1) - \mathcal{F}(N) = E_F$$

EQUALITY OF THE ELECTROCHEMICAL POTENTIAL OF DOTS AND LEADS

THE FREE ENERGY OF THE DOT AT $T=0$:

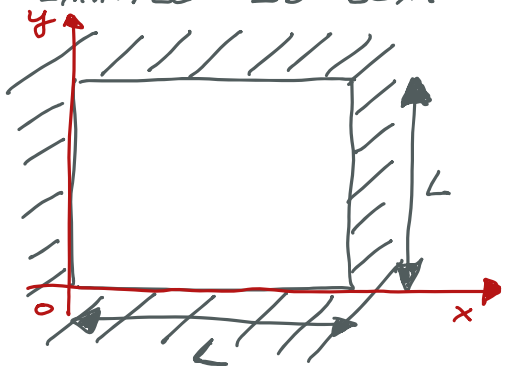
$$\mathcal{F}(N) = \sum_{p=1}^{N} E_p + U(N)$$

SINGLE ELECTRON ENERGY

ELECTRON-ELECTRON INTERACTION

SINGLE ELECTRON ENERGY

EXAMPLE 2D-BOX:



$$H(x,y)\,\psi(x,y) = E\,\psi(x,y)$$

$V(x,y) = 0 \qquad 0 < x, y < L$

$V(x,y) = \infty \qquad$ OTHERWISE

$$\Rightarrow H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} \qquad 0 < x, y < L$$

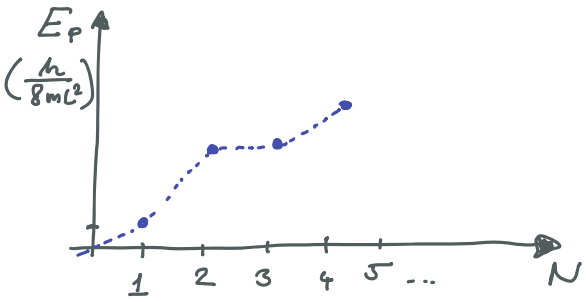
$\psi(x=0/L,\ y=0/L) = 0.$

$$-\frac{\hbar^2}{2m}\left(\partial_x^2 + \partial_y^2\right)\psi(x,y) = E\,\psi(x,y)$$

$$\psi(x,y) = A \sin(k_x \cdot x) \sin(k_y \cdot y)$$

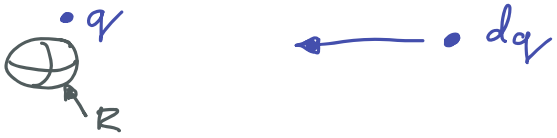
$$k_{x,y} = n_{x,y} \cdot \frac{\pi}{L}$$

$$E_{n_x n_y} = \frac{h}{8mL^2}\left(n_x^2 + n_y^2\right)$$



ELECTRON – ELECTRON INTERACTION

CLASSICAL MODEL : "ORTHODOX MODEL"



POTENTIAL : $\phi(Q) = \frac{W}{e} = \frac{Q}{C}$ ASSUMING C IS INDEPENDENT OF N

$$\Delta W = -\int_{-\infty}^{R} \Delta q\, E(q)\, dr = -\Delta q\, \phi(q) = -\Delta q \frac{q}{C}$$

$$U = -\sum_{\Delta q_i} \Delta W \simeq \int dq \frac{q}{C} = \frac{(Ne)^2}{2C}$$

ADDITIONAL EXTERNAL POTENTIAL:

$$U = \frac{(Ne)^2}{2C} - Ne\,\phi_E =$$

$$= (Ne - Q_E)^2/2C + \text{const.} \qquad Q_E = C\phi_{EXT}$$

"EXTERNALLY INDUCED CHARGE"

PARABOLIC POTENTIAL

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}m\omega_0^2(x^2+y^2)$$

SOLUTION: $E_{n,l} = \hbar\omega_0(2n + |l| + 1)$

n: NUMBER OF NODES RADIALLY

l: NUMBER OF NODES CIRCUMFERENTIALLY

s: SPIN QUANTUM NUMBER WITH $s = \pm\frac{1}{2}$

IF THERE IS MAGNETIC FIELD IN THE Z-DIRECTION, (WE CAN IGNORE ZEEMAN SPLITTING HERE):

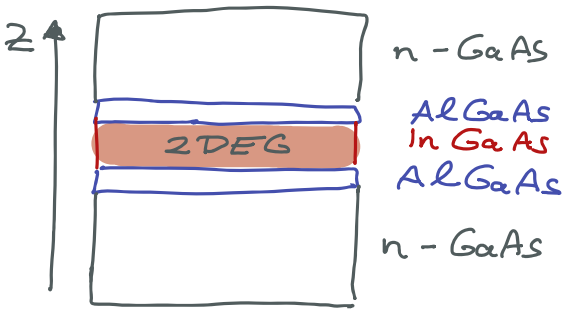
$$E_{n,l} = \hbar\tilde{\omega}_0(2n + |l| + 1) - \frac{1}{2}\hbar l\omega_c,$$

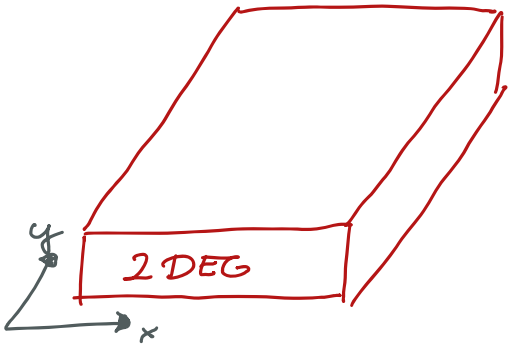
$$\omega_c = \frac{eB}{mc}$$

$$\omega_0 = \sqrt{(\omega_c/2)^2 + \omega_0^2}$$

FOCK-DARWIN SPECTRA

HETEROSTRUCTURE:





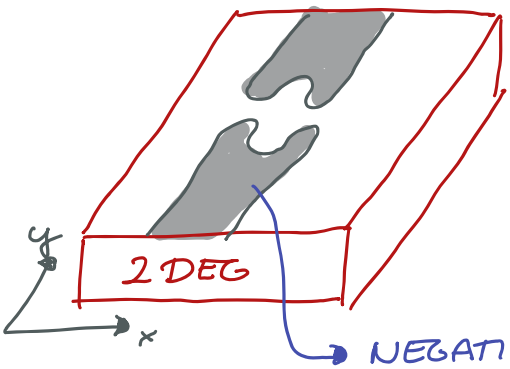
VERTICAL DOT

GATE VOLTAGE PUSHES ELECTRONS IN

MAKE A PILLAR

LATERAL DOT

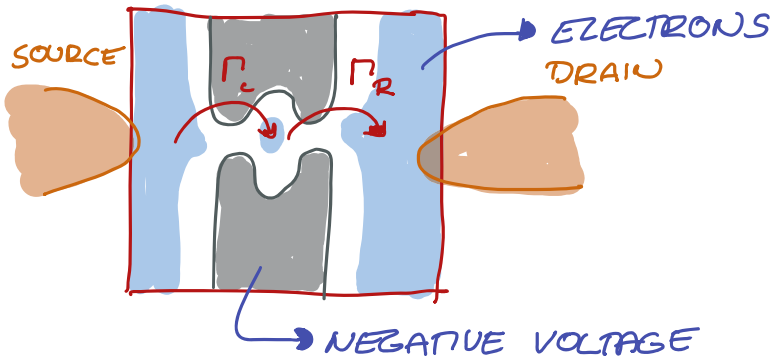
PUT GATES ON TOP



NEGATIVE VOLTAGE → DEPLETE

ELECTRONS

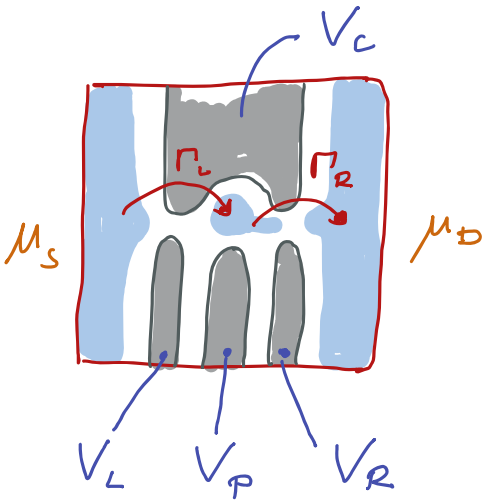
TOP VIEW:



THE SAME GATE CONTROLS THE ELECTRON DENSITY + TUNNELING RATE

↓

MORE GATES → MORE CONTROL



FOR EXAMPLE $\Gamma_R >> \Gamma_L$

MEASURING THE QUANTUM DOT:

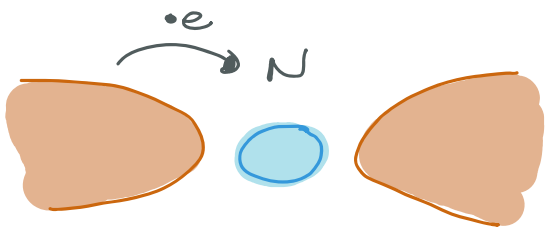
$$Q = Ne$$

TOTAL ENERGY OF THE DOT

$$E(N) = \underbrace{\frac{Q^2}{2C} - QV_G}_{\frac{e^2}{2C}\left[N - \frac{CV_G}{e}\right]^2} + \sum_i E_{ORB}(i)$$

ENERGY OF ADDING 1 MORE ELECTRON

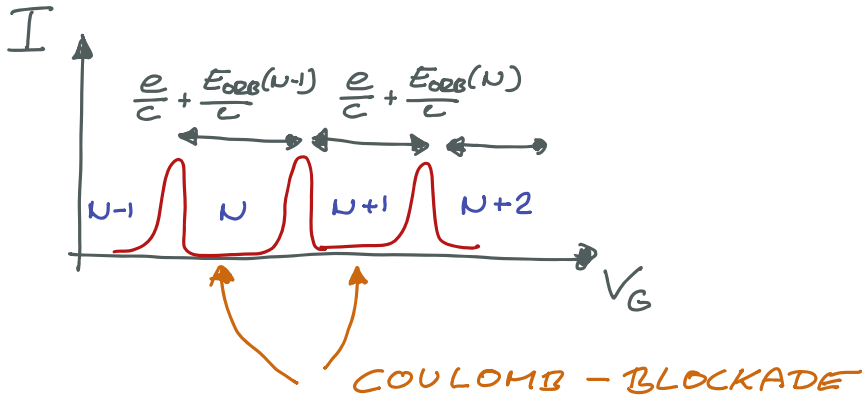
$$\mu(N) = E(N+1) - E(N) = \frac{e^2}{C} \cdot N - eV_G + E_{ORB}(N)$$



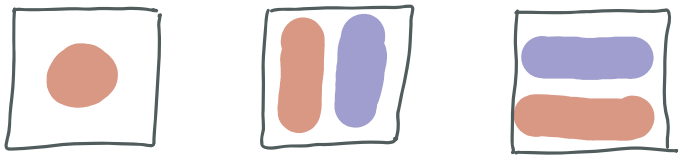
WE CAN MEASURE CURRENT IF:

$$E(N+1) = E(N) \qquad \text{OR} \qquad \mu(N) = 0$$

$$V_G = -\frac{e}{C}\cdot N + \frac{E_{ORB}(N)}{e}$$



SQUARE DOT:



$\psi_0(x,y)$ $\psi_1(x,y)$ $\psi_2(x,y)$

$$\psi(x,y) \propto \sin\left(n_x \frac{\pi}{L}\cdot x\right) \sin\left(n_y \frac{\pi}{L}\cdot y\right)$$

$$E = \underbrace{\frac{\hbar^2}{8mL^2}}_{E_0}\left(n_x^2 + n_y^2\right)$$

$E_0 \rightarrow n_x = 1 \quad n_y = 1 \qquad E_0 = 2\varepsilon_0$

$E_1 \rightarrow n_x = 1 \quad n_y = 2 \qquad E_1 = 5\varepsilon_0$

$E_2 \rightarrow n_x = 2 \quad n_y = 1 \qquad E_2 = 5\varepsilon_0$

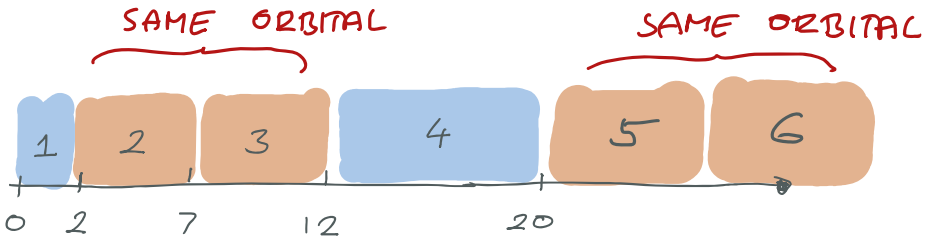
DEGENERATE

$E_3 \rightarrow n_x = 2 \quad n_y = 2 \qquad E_3 = 8\varepsilon_0$

$E_4 \rightarrow n_x = 3 \quad n_y = 2 \qquad E_4 = 11\varepsilon_0$

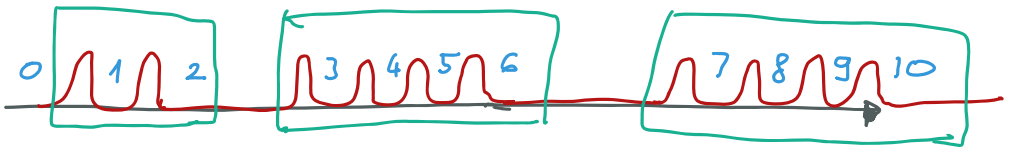
$E_5 \rightarrow n_x = 2 \quad n_y = 3 \qquad E_5 = 11\varepsilon_0$

$\vdots$

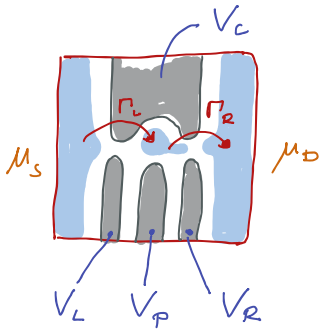


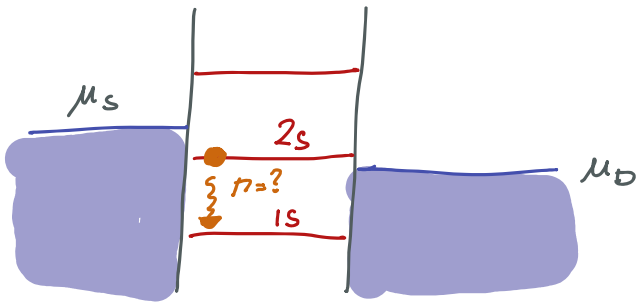
SPIN DEGENERACY: X2

ASSUME $\frac{e^2}{C} << \varepsilon_0$



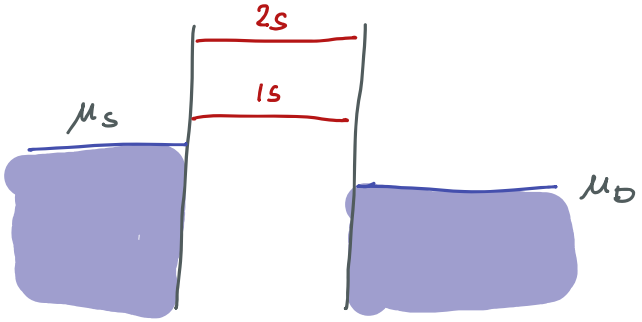
ARTIFICIAL HYDROGEN ATOM LIFETIME





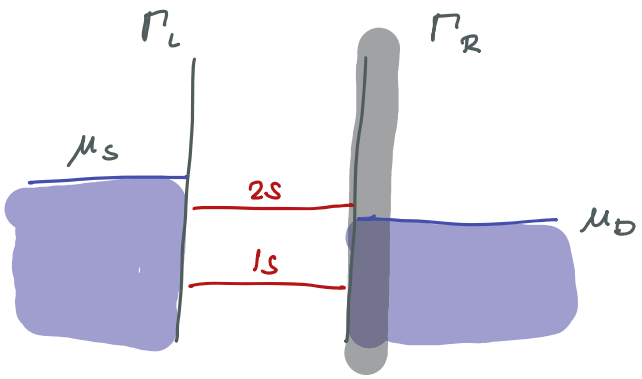
THE FIRST T_1 MEASUREMENT:

INITALIZE:



NO ELECTRONS IN THE DOT.

PUT ELECTRON INTO THE EXCITED STATE:



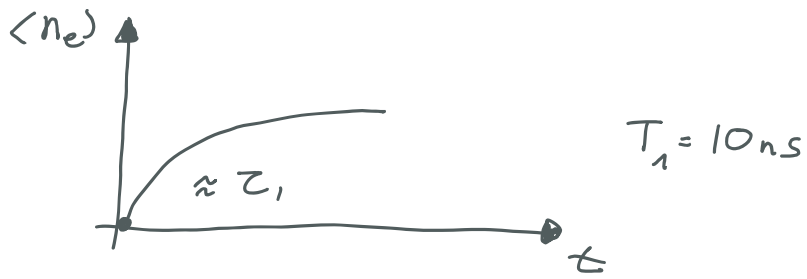
$\Gamma_R \ll \Gamma_L$

CASE 1: ELECTRON GOES INTO 1s

→ NO CURRENT

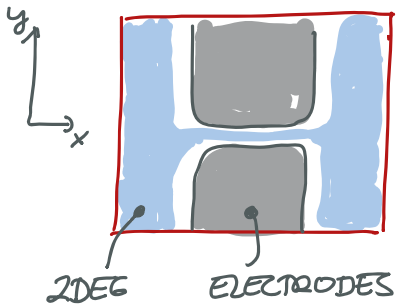
CASE 2: ELECTRON GOES INTO 2s

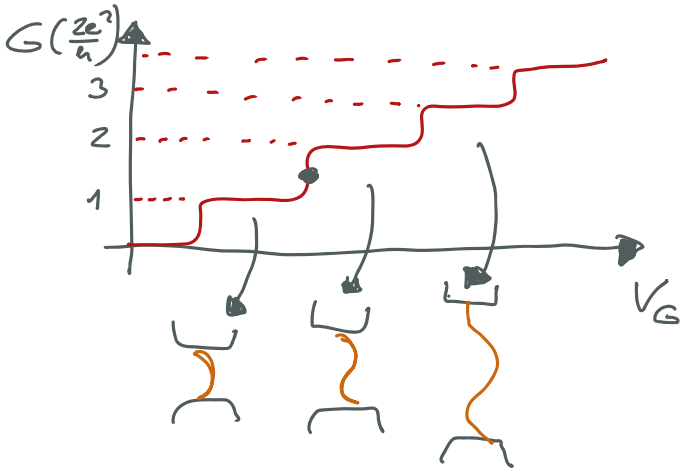
→ CURRENT IF IT DID NOT DECAY



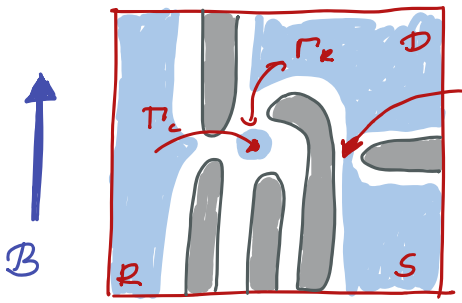
$T_1 = 10\,ns$

SENSITIVE QUBIT READ-OUT – QPC





THE CONDUCTANCE IS SENSITIVE TO THE VOLTAGE AT CERTAIN VALUES



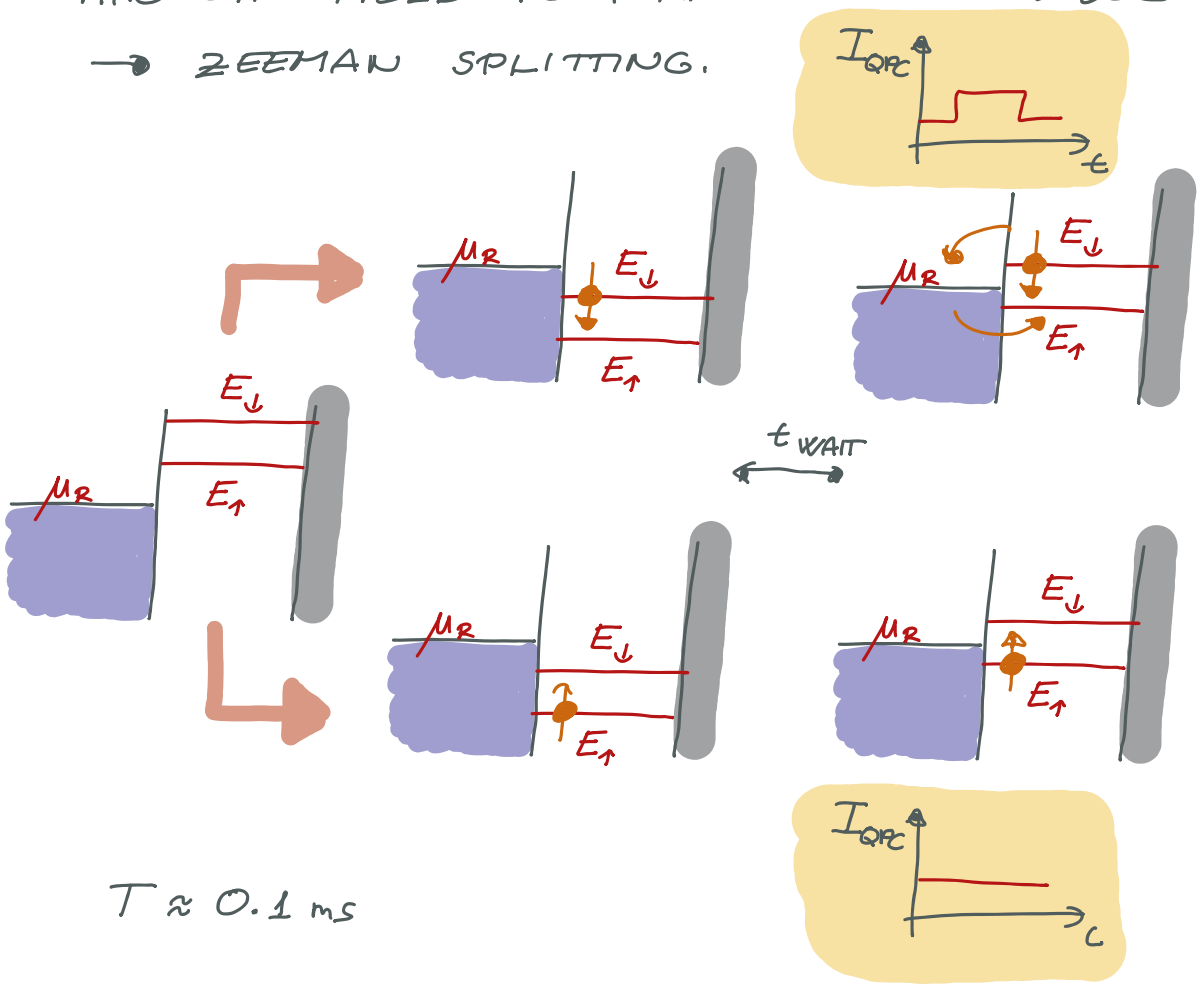
QPC : MEASURING THE ELECTROSTATIC ENVIRONMENT

NOT MEASURING CURRENT ACROSS THE DOT

→ LESS INVASIVE

MAGNETIC FIELD IN-PLANE OF THE 2DEG

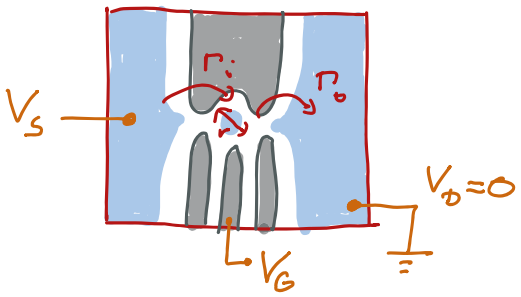
→ ZEEMAN SPLITTING.



$T \approx 0.1$ ms

SO FAR TWO TYPES OF DEVICES ON 2DEG:

QUANTUM DOT

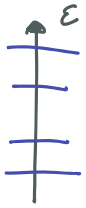


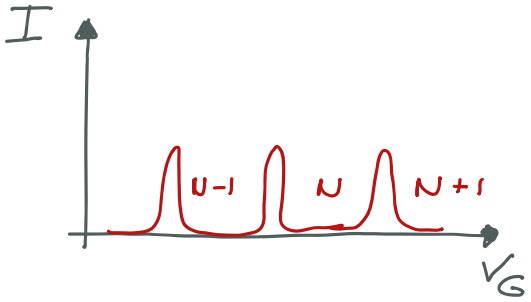
0-DIMENSIONAL

$$H = -\frac{\hbar^2}{2m}(\partial_x^2+\partial_y^2) + \frac{1}{2}m\omega_0^2(x^2+y^2)$$

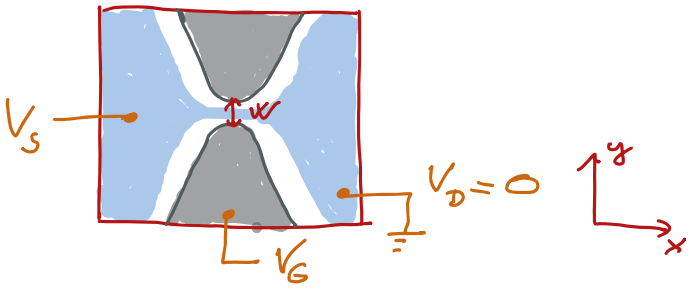
$$\psi(x,y) \propto e^{-(x^2+y^2)/r^2}$$

$$\mathcal{E}(n,l) = \hbar\omega_0(2n+|l|+1)$$





QUANTUM POINT CONTACT



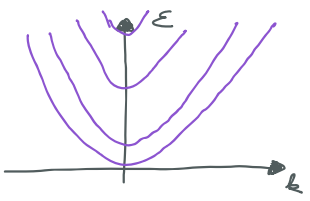
1-DIMENSIONAL

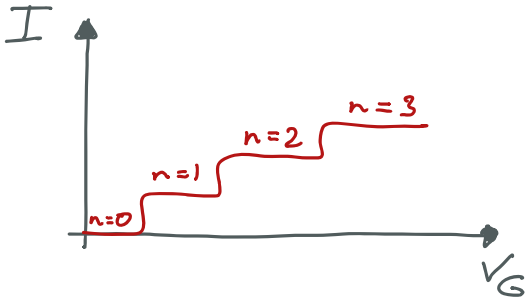
$$H = -\frac{\hbar^2}{2m}(\partial_x^2+\partial_y^2)$$

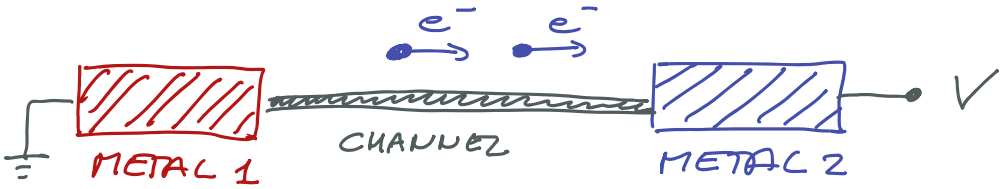
+ B.C. $V(x,y=0) = V(x,y=W) = 0$

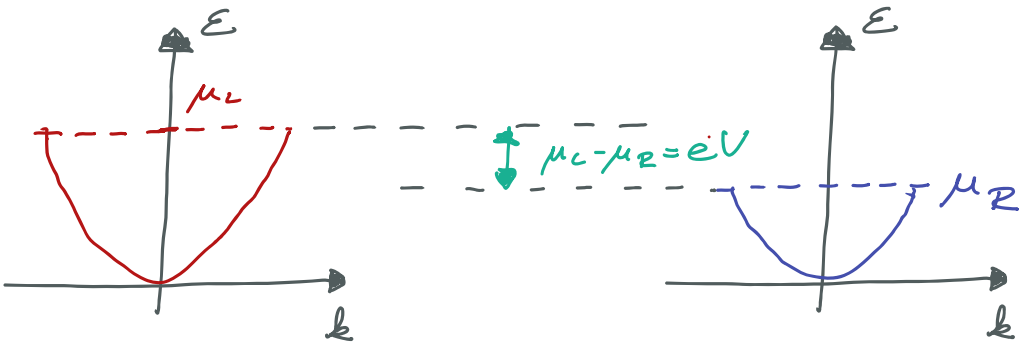
$$\psi(x,y) \propto e^{ikx}\sin\left(n\frac{\pi}{W}y\right)$$

$$\mathcal{E}(n,k) = \frac{\hbar^2k^2}{2m} + n^2\cdot\frac{\pi^2}{W^2}$$



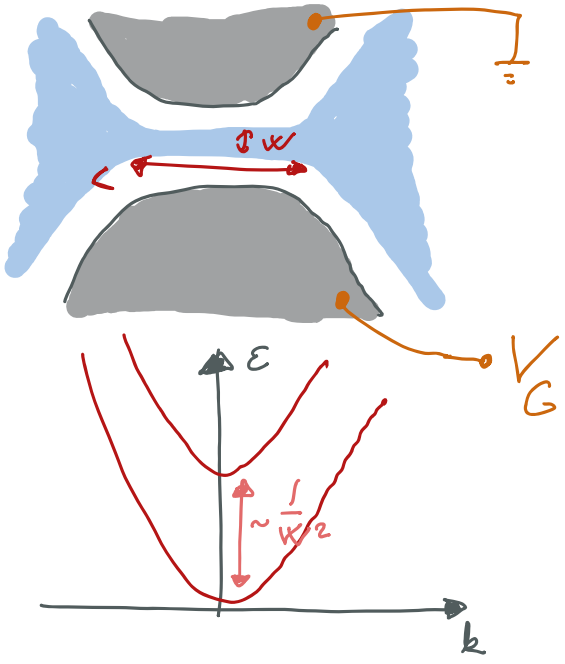




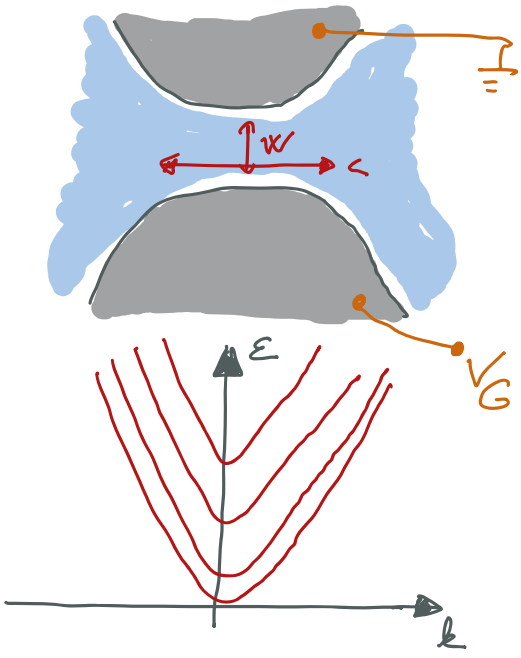


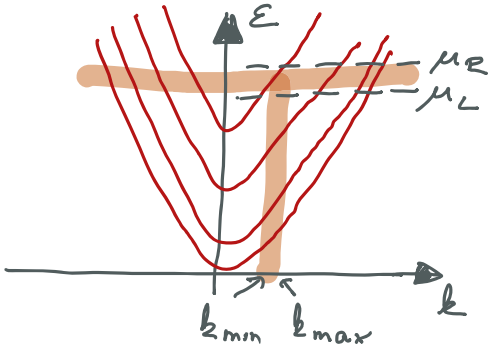
APPLYING POSITIVE VOLTAGE ON A METAL: IT IS EASIER TO PUT AN ELECTRON AT THE HIGHEST ENERGY.

$V_G << 0 \rightarrow W$ SMALL $\rightarrow n^2\frac{\pi^2}{W^2}$ LARGE



$V_G >> 0$ W IS BIG





$$v = \frac{1}{\hbar}\frac{\partial E}{\partial k}$$

$$\Delta k \times \frac{1}{\Delta k} \qquad \frac{L}{2\pi}$$

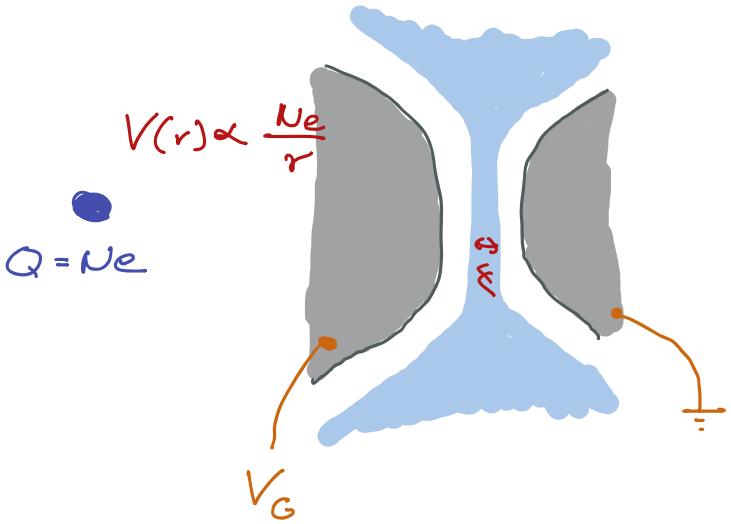
$$I = \frac{e}{\Delta t} = \sum_{modes} \sum_{k_{min}}^{k_{max}} 2\,\frac{e}{L/v(k)} = \sum_{modes} \int dk\, \frac{L}{2\pi}\, 2\,\frac{e}{L/v(k)} =$$

$$= \sum_{modes} \int_{k_{min}}^{k_{max}} dk\, \frac{1}{\pi}\, e\, v(k) =$$

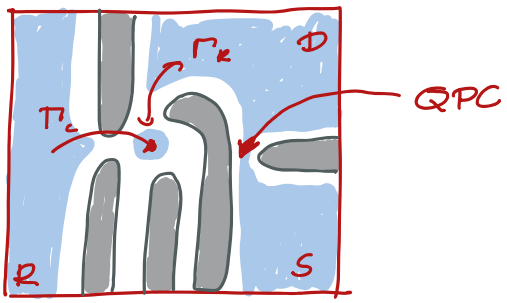
$$= \sum_{modes} \int_{k_{min}}^{k_{max}} dk\, \frac{1}{\pi}\, e\, \frac{1}{\hbar}\, \frac{\partial E}{\partial k} =$$

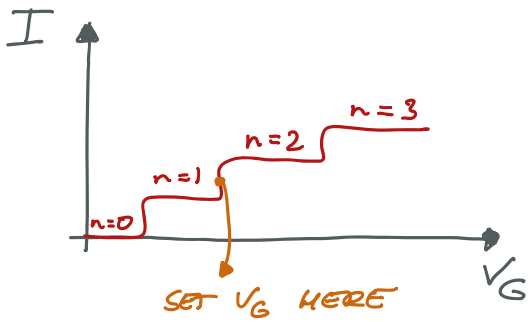
$$= \sum_{modes} \int_{0}^{eV} dE\, \frac{e}{\pi \frac{\hbar}{2\pi}} = \sum_{modes} \frac{2e^2}{h} \cdot V = \boxed{\frac{2e^2}{h} \cdot M \cdot V} \qquad G$$

QUBIT READ-OUT

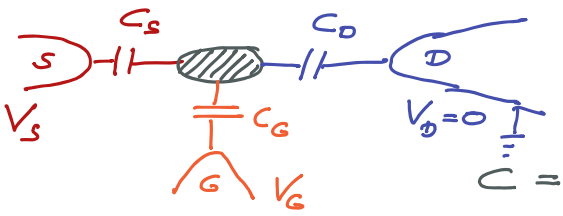


THE TOTAL GATE VOLTAGE : $V_G + V(r)$





COULOMB DIAMONDS



$$C = C_S + C_D + C_G$$

$$\left[\begin{aligned} E &= -QV_G + Q^2/2C = \\ &= -Ne\cdot V_G + (Ne)^2/2C \\ &= (Q-Q_0)^2/2C + \text{cost.} \\ Q_0 &= CV_G \end{aligned}\right]$$

THE TOTAL ENERGY OF THE DOT:

$$E_{DOT}(N) = (Ne - C_S V_S - C_G V_G)^2/2C + \sum_{p=1}^{N} E_p$$

THE ELECTROCHEMICAL POTENTIAL:

$$\mu(N) = E_{DOT}(N) - E_{DOT}(N-1) =$$

$$= \left\{ [Ne - C_S V_S - C_G V_G]^2 - [(N-1)e - C_S V_S - C_G V_G]^2 \right\} \times \frac{1}{2C} + E_N =$$

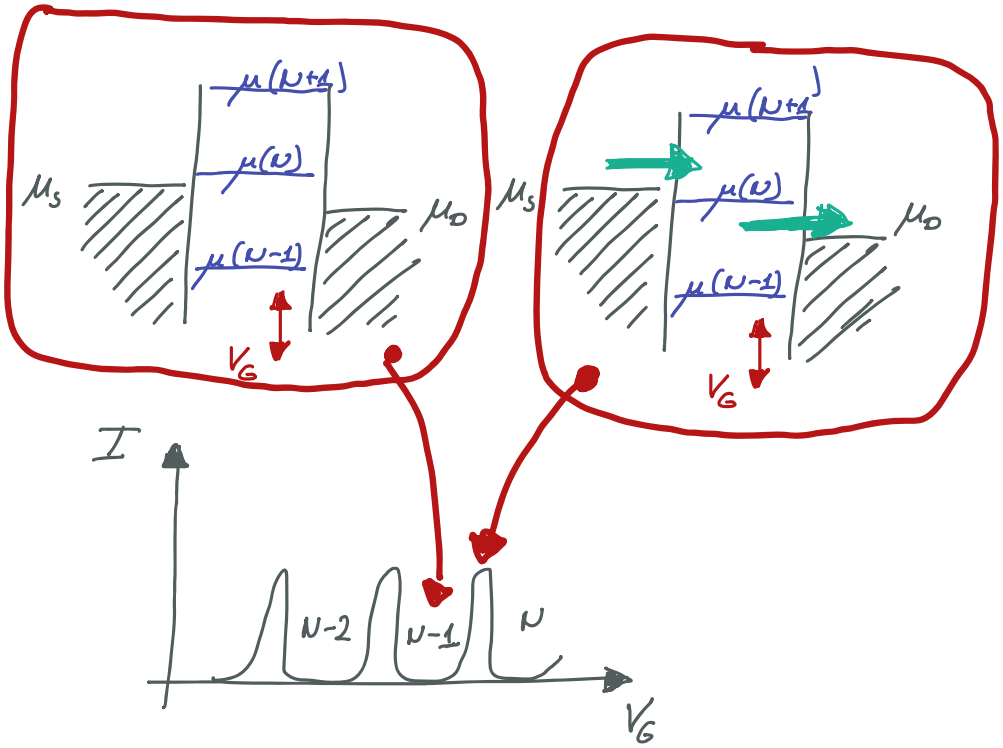
$$= \left[N - \frac{1}{2}\right] \underbrace{\frac{e^2}{C}}_{E_C} - e\left[C_S V_S + C_G V_G\right]/C + E_N$$

$$\boxed{\mu(N) = \left(N - \frac{1}{2}\right) E_C - \frac{C_S}{C} eV_S - \frac{C_G}{C} eV_G + E_N}$$

- $\left(N - \frac{1}{2}\right) E_C$ — INTERACTION WITH OTHER ELECTRONS
- $\frac{C_S}{C} eV_S + \frac{C_G}{C} eV_G$ — MOVING CHARGE TO A POTENTIAL
- E_N — SINGLE ELECTRON EXCITATIONS

LOW-BIAS REGIME

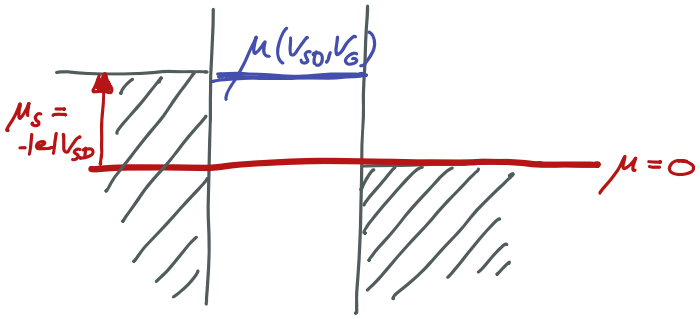
BIAS $V_{SD} = V_S - V_D \; (= V_S)$ FIXED



HIGH-BIAS REGIME

$N = FIX \qquad V_D = 0 \;,\; \mu_D = 0$

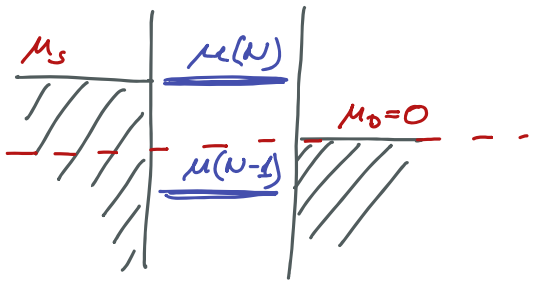
$$\mu(V_{SD}, V_G) = \mu^{\circ}(N) - \frac{C_S}{C}|e|V_{SD} - \frac{C_G}{C}|e|V_G$$



WHAT ARE THE CONDITIONS TO HAVE N-1 ELECTRONS ON DOT

$V_{SD} < 0$.

$\mu(N) > \mu_S$ & $\mu(N-1) < \mu_D$



(1) "UPPER" BORDER BETWEEN $N-1 \leftrightarrow N$

$$\mu_S(V_{SD}) \leq \mu(N, V_{SD}, V_G)$$

$$-|e|V_{SD} \leq \mu^\circ(N) - \frac{C_S}{C}|e|V_{SD} - \frac{C_G}{C}|e|V_G$$

$$V_{SD} \geq -\mu^\circ(N)/|e| + \frac{C_S}{C}V_{SD} + \frac{C_G}{C}V_G$$

$$\boxed{V_{SD} \geq (N-\tfrac{1}{2})\frac{|e|}{C-C_S} + \frac{C_G}{C-C_S}\cdot V_G}$$

$V_{SD} \simeq 0$

$V_G = (N-\frac{1}{2})\frac{|e|}{C_G}$

(2) "LOWER" BORDER BETWEEN $N-1 \leftrightarrow N$

$$0 \geq \mu(N-1, V_{SD}, V_G)$$

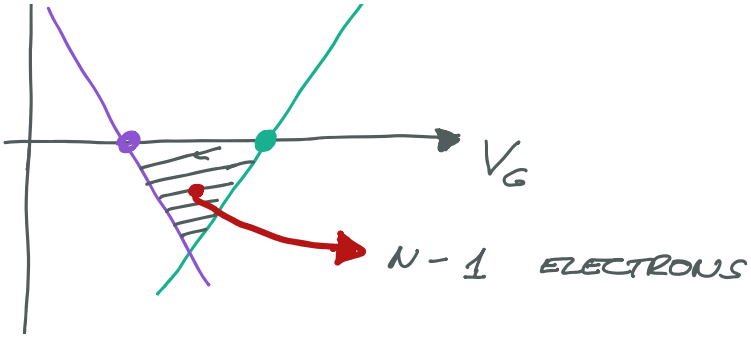
$$0 \leq -(N-3/2)|e| + C_S V_{SD} + C_G V_G$$

$$\boxed{V_{SD} \geq (N-3/2)|e|/C_S - \frac{C_G}{C_S}V_G}$$

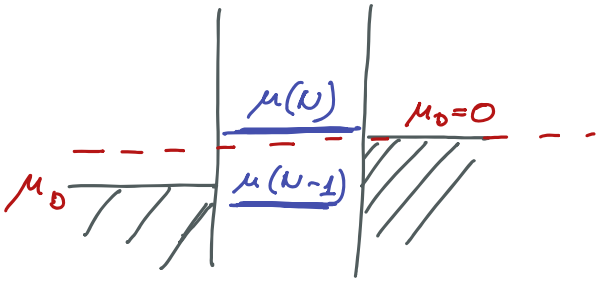
$V_{SD} \simeq 0$

$V_G = (N-\frac{3}{2})\frac{|e|}{C_G}$

V_{SD} ▲



$V_{SD} > 0$



(1) "UPPER" BORDER BETWEEN $N-1 \leftrightarrow N$

$$0 \leq \mu(N, V_{SD}, V_G)$$

$$0 \leq \left(N - \tfrac{1}{2}\right)\frac{|e|}{C} - \frac{C_S}{C}V_{SD} - \frac{C_G}{C}V_G$$

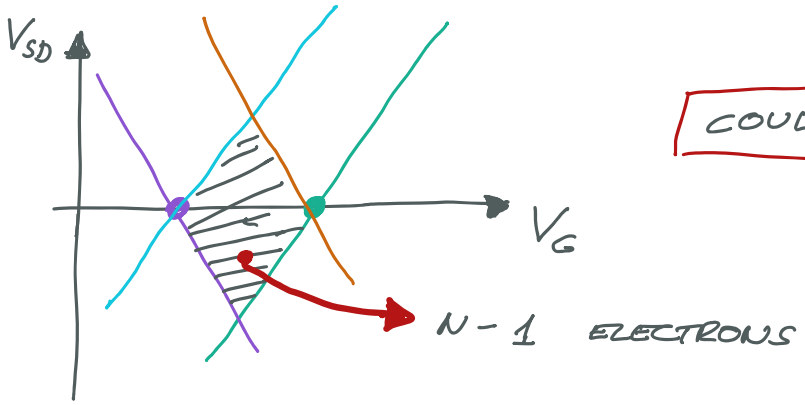
$$\boxed{V_{SD} \leq \left(N - \tfrac{1}{2}\right)\frac{|e|}{C_S} - \frac{C_G}{C_S}V_G}$$

(2) "LOWER" BORDER BETWEEN $N-1 \leftrightarrow N$

$$\mu_D \geq \mu(N-1, V_{SD}, V_G)$$

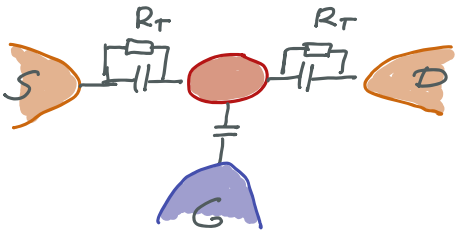
$$V_{SD} \leq -\left(N - \tfrac{3}{2}\right)\frac{|e|}{C} + \frac{C_S}{C}V_{SD} + \frac{C_G}{C}V_G$$

$$\boxed{V_{SD} \leq -\left(N - \tfrac{3}{2}\right)\frac{|e|}{C-C_S} + \frac{C_G}{C-C_S}V_G}$$



COULOMB - DIAMONDS

HOW MANY ELECTRONS ARE ON THE DOT?

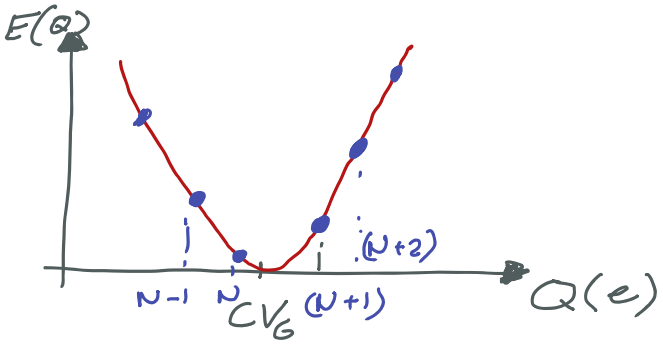


LET'S IGNORE ORBITAL EFFECTS

$$Q = Ne$$

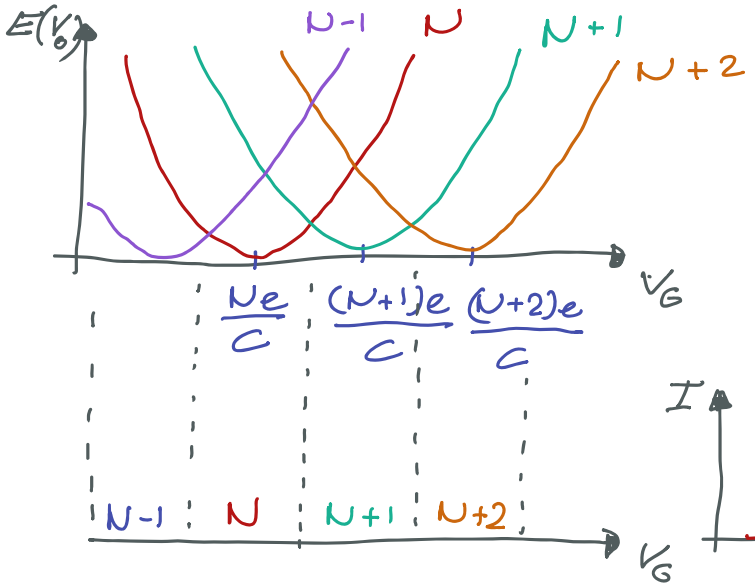
$$E = \frac{Q^2}{2C} - QV_G =$$

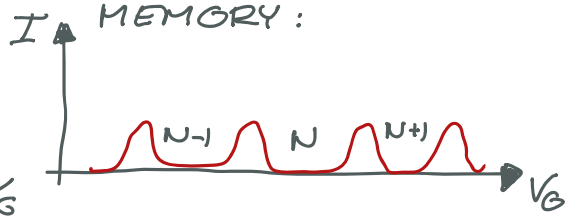
$$= \frac{(Q - CV_G)^2}{2C} + \text{const.}$$



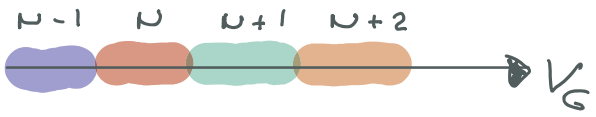
OTHER WAY TO LOOK AT IT:

HOW DOES THE ENERGY OF N ELECTRONS DEPEND ON THE GATE VOLTAGES?



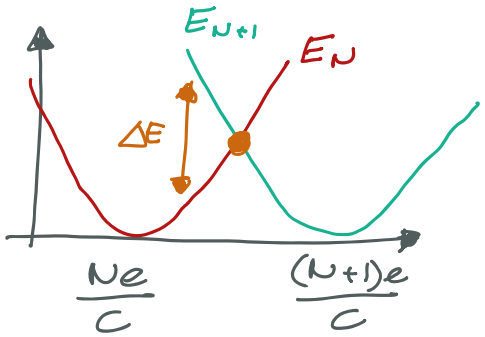


STABILITY DIAGRAM: THE NUMBER OF ELECTRONS IN THE DOT



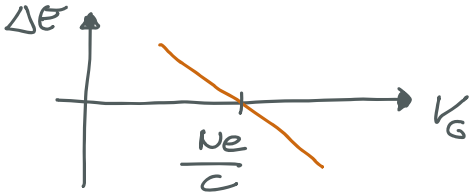
↳ IT WILL BE MORE INTERESTING IN DOUBLE DOTS

WHAT IS THE ENERGY OF THE EXCITED STATE CLOSE TO $\frac{(N+\frac{1}{2})e}{C}$?

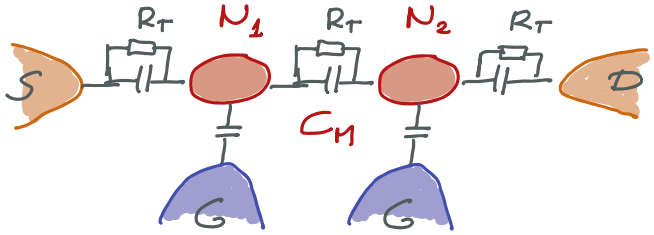


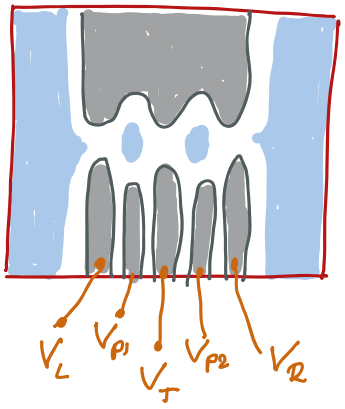
$$E_N = \frac{(Ne - CV_G)^2}{2C} \qquad E_{N+1} = \frac{[(N+1)e - CV_G]^2}{2C}$$

$$\Delta E = E_{N+1} - E_N = -eV_G + \frac{Ne}{C}$$



DOUBLE DOTS





$$E_{TOT} \simeq \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} + \boxed{\frac{1}{2}\frac{Q_1 Q_2}{C_M}} - Q_1 V_{G1} - Q_2 V_{G2}$$

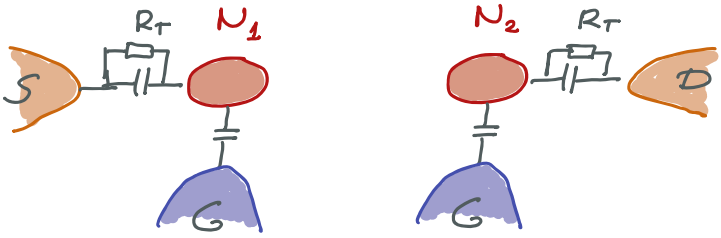
COUPLING BETWEEN DOTS $E_{COUPLING}$

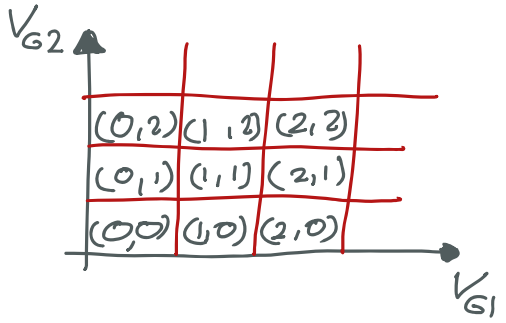
MORE PRECISELY:

$$U = \frac{1}{2} E_{C1} N_1^2 + \frac{1}{2} E_{C2} N_2^2 + N_1 N_2 E_{CM} +$$
$$- \frac{1}{|e|}\left(C_{G1} V_{G1} \left(N_1 E_{C1} + N_2 E_{CM} \right) + \quad 1 \leftrightarrow 2 \right.$$

STABILITY DIAGRAM

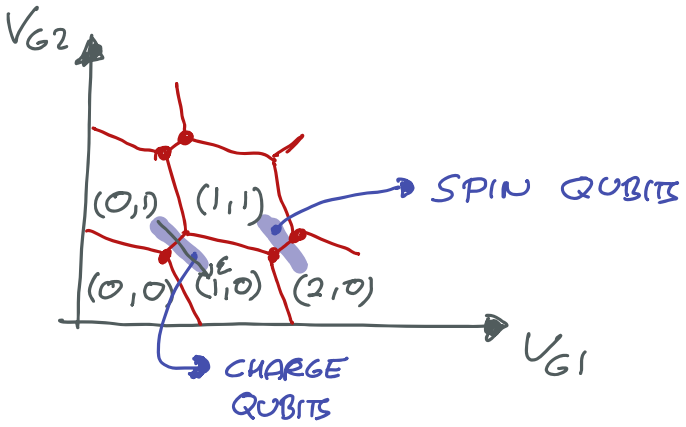
UNCOUPLED DOTS





(N_1, N_2)

NOW, IF WE TURN ON THE INTERACTION

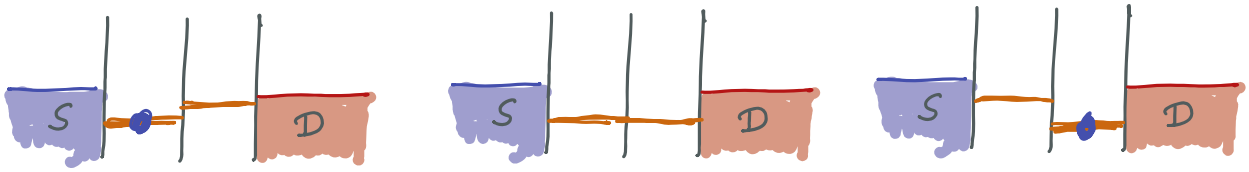


$$\varepsilon = e\left[V_{G1} - V_{G2}\right]$$

SEMI CONDUCTOR-BASED QUBITS

CHARGE QUBITS

$V_{G1} > V_{G2}$ $\qquad$ $V_{G1} = V_{G2}$ $\qquad$ $V_{G1} < V_{G2}$



$|L\rangle$ $\qquad$ WHERE IS THE ELECTRON? $\qquad$ $|R\rangle$

$$\Delta E_{L-R} = \varepsilon$$

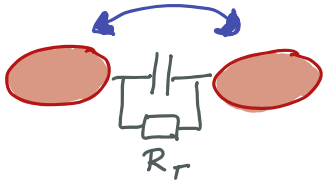
$$\rightarrow \quad H = \begin{bmatrix} -\varepsilon/2 & 0 \\ 0 & \varepsilon/2 \end{bmatrix} = \qquad \text{BASIS} \quad \{ |L\rangle, |R\rangle \}$$

$$= -\frac{\varepsilon}{2} |L\rangle\langle L| + \frac{\varepsilon}{2} |R\rangle\langle R|$$

$$H|L\rangle = -\frac{\varepsilon}{2}$$
$$H|R\rangle = \frac{\varepsilon}{2}$$

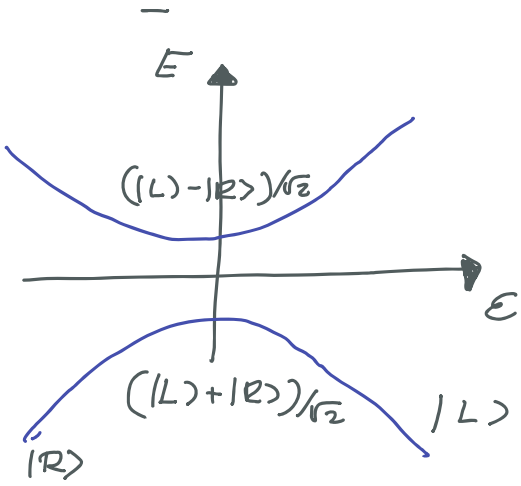
$$H = -\frac{\varepsilon}{2} \sigma_z \, .$$

TUNNELING EVENTS



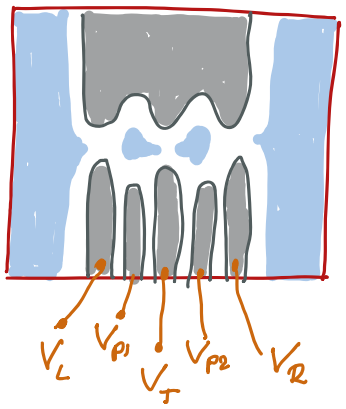
$$H_T = t \underbrace{\left[|L\rangle\langle R| + |R\rangle\langle L| \right]}_{\sigma_x}$$

$$H = -\frac{\varepsilon}{2} \sigma_z + \Delta \sigma_x$$



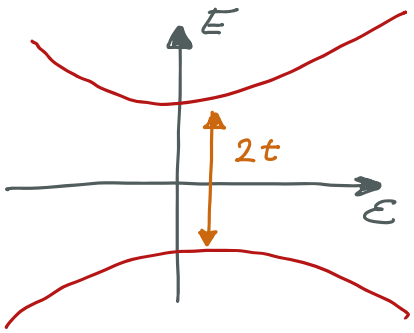
$$E_{1,2} = \pm\sqrt{(\varepsilon/2)^2 + t^2}$$

IF V_T NOT TOO NEGATIVE

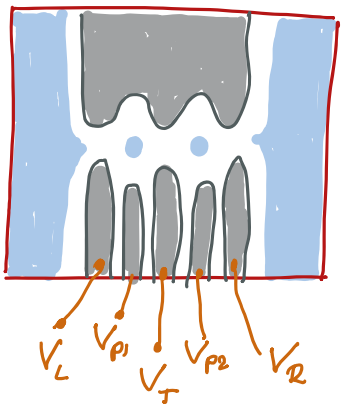


t IS LARGE

$|\psi_L(r)|^2 \, |\psi_R(r)|^2 \gg 0$

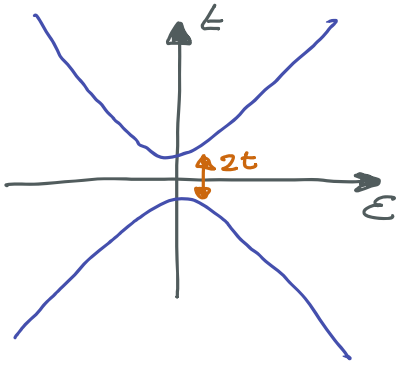


IF V_T IS VERY NEGATIVE



t IS SMALL

$|\psi_L(r)|^2 \, |\psi_R(r)|^2 \approx 0$

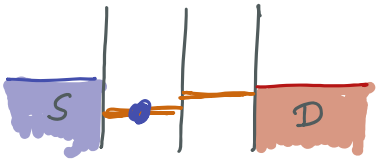


t TUNES THE CROSSOVER FROM LOCALIZED TO DELOCALIZED STATE

NON-ADIABATIC CONTROL

1. INITIALIZATION

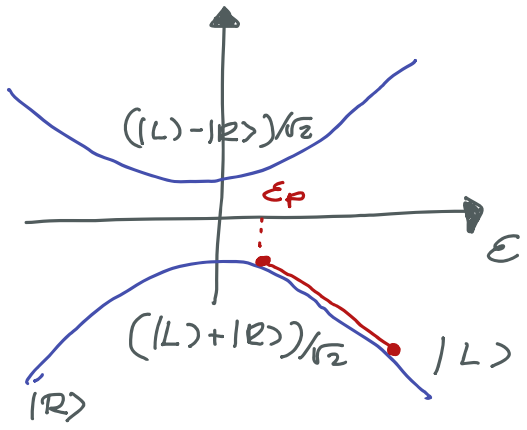
$\varepsilon \gg 0$

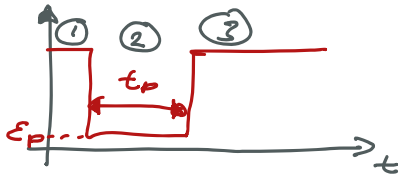


$|\psi_0\rangle = |L\rangle$

$|L\rangle$

2. FAST GATE:





FOR EXAMPLE, $\varepsilon_p = 0. \rightarrow H = \Delta \sigma_x$

$\rightarrow U = e^{-iHt_p}$

$R_x(\theta = 2 \cdot \Delta t) = e^{-i \Delta t \sigma_x}$

↳ x ROTATION

IF $\varepsilon_p \neq 0$ $\quad H = \Delta\sigma_x - \varepsilon/2\,\sigma_z$

$U = e^{-i(\Delta\sigma_x - \varepsilon/2\sigma_z)t}$

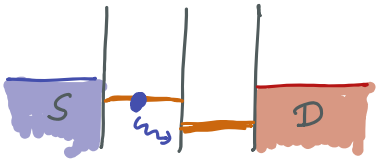
↳ ROTATION AROUND DIFFERENT AXIS

(3.) READ OUT

MEASURE THE POPULATION OF THE LEFT DOT

$M_L = |L\rangle\langle L|$

T_1 - MEASUREMENT



$|R\rangle$

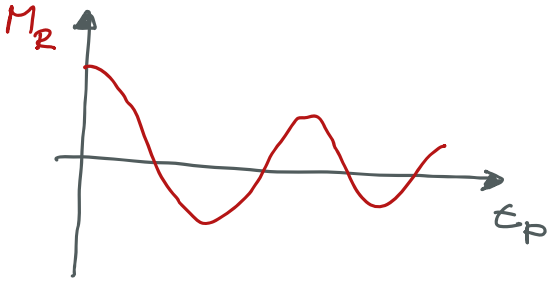
WHERE DOES THE ENERGY GO ?

→ PHONONS

$T_1 \approx 30\,ns$

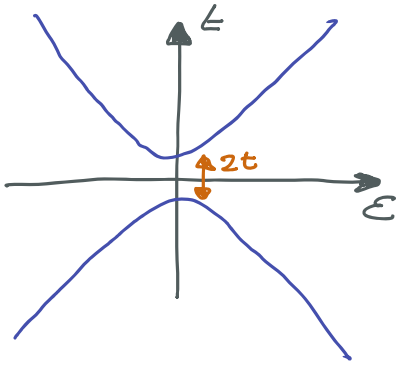
T_2 - MEASUREMENT

DECAY OF DRIVEN OSCILLATION



$T_2 \approx 7\,ns$

CHARGE NOISE : $H = -\frac{\varepsilon(t)}{2}\sigma_z + t(t)$



$$E(t) = \sqrt{[\varepsilon(t)]^2 + 4t(t)^2}$$

↓

QUBIT FREQUENCY CHANGES

↓

PURE DEPHASING

BEST T_φ → @ $\varepsilon = 0$ FIRST-ORDER INSENSITIVE SWEET SPOT

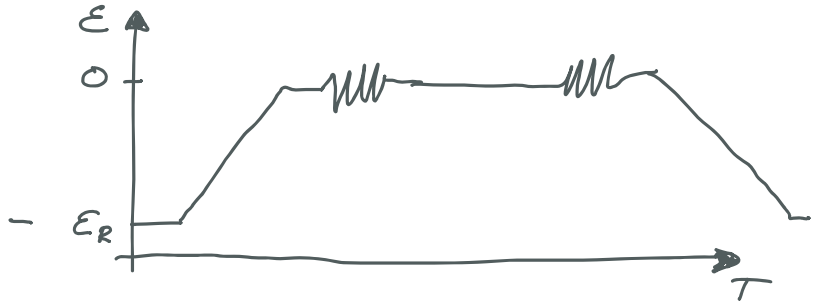
① ADIABATIC TRANSFER OF $|R\rangle \rightarrow (|R\rangle + |L\rangle)/\sqrt{2}$

② $X_{\pi/2}$ ROTATION

③ WAIT TIME ϵ

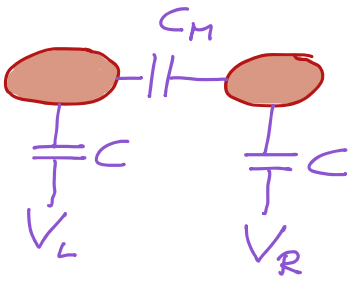
④ $X_{\pi/2}$ ROTATION

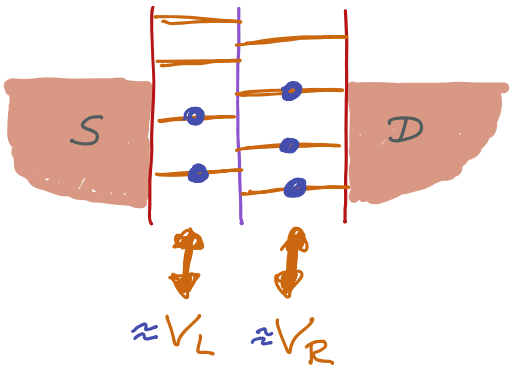
⑤ PROJECTING BACK TO $|R\rangle$



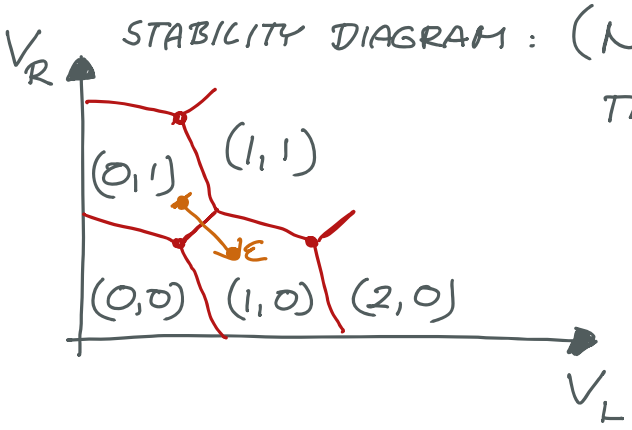
CHARGE QUBIT

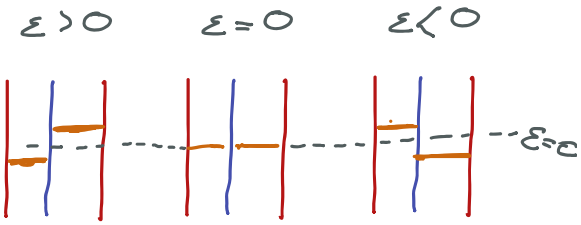
DOUBLE QUANTUM DOT:

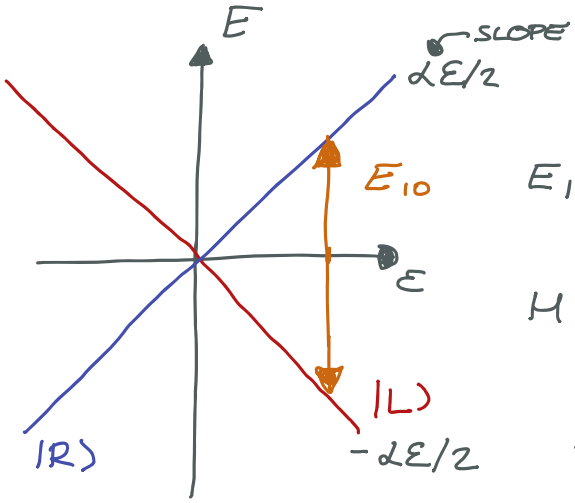




STABILITY DIAGRAM: (N_L, N_R) ELECTRONS ON THE DOT





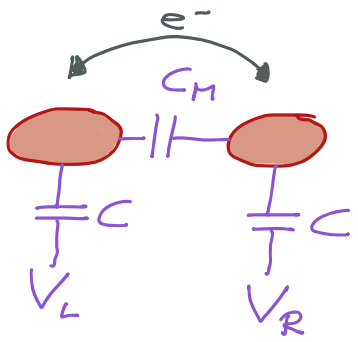


$$E_{10} = \frac{1}{2}\alpha\cdot\varepsilon - \left(-\alpha\frac{1}{2}\right)\varepsilon = \alpha\cdot\varepsilon$$

$$H = \frac{1}{2}\left[\alpha\cdot\varepsilon\,|R\rangle\langle R| + (-\alpha\varepsilon)|L\rangle\langle L|\right] =$$

$$= -\frac{1}{2}\alpha\cdot\varepsilon\,\sigma_z$$

IN $\begin{bmatrix} |L\rangle \\ |R\rangle \end{bmatrix}$ BASIS

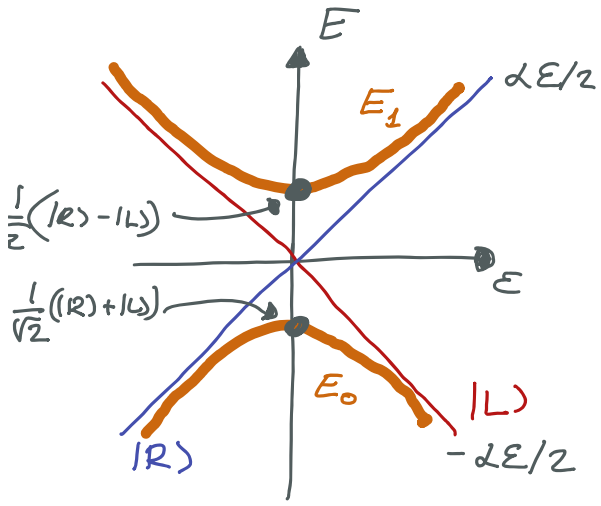


ELECTRONS JUMP:

$$H = -|t|\left[\,|L\rangle\langle R| + |R\rangle\langle L|\,\right] =$$

$$= -|t|\,\sigma_x$$

$$H = -\frac{1}{2}\alpha\mathcal{E}\,\sigma_z - |t|\,\sigma_x$$



CHARGE HYBRIDIZATION

$$E_{0,1} = \mp\sqrt{\left(\frac{1}{2}\alpha\mathcal{E}\right)^2 + |t|^2}$$

AT $\mathcal{E} = 0$

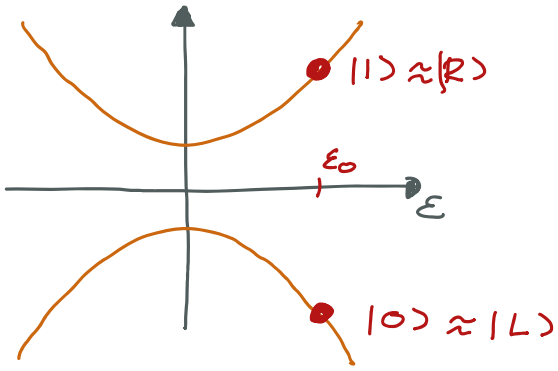
$$E_{0,1} = \mp t \qquad \Delta E_\alpha = 2t$$

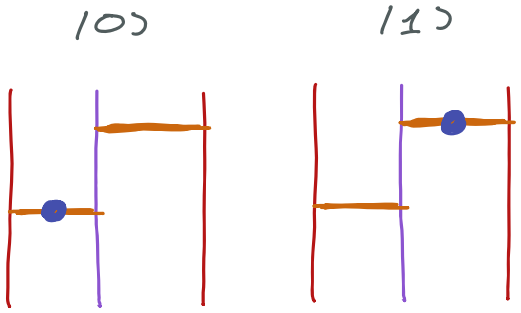
$$H = \begin{bmatrix} 0 & -t \\ -t & 0 \end{bmatrix}$$

$$\psi_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad E_0 = -t$$

$$\psi_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad E_1 = t$$

WE FIX $\varepsilon = \varepsilon_0 >> 0$, AND DEFINE THE QUBIT THERE:





HOW TO DO A GATE?

FOR EXAMPLE $R_x(\pi)$ [X GATE]

① $t=0$ $\varepsilon = \varepsilon_0$ $H \approx -\frac{1}{2}\alpha\varepsilon_0\sigma_z$ $|0\rangle$

② MOVE ε FAST (NON-ADIABATIC) TO $\varepsilon = 0$
($\hbar/t <$)

$t = 0.1\,ns$ $|\psi\rangle \approx |0\rangle$ $H = -|t|\sigma_x$

$U = e^{i|t|\sigma_x/\hbar \cdot t}$ $|\psi(t)\rangle = U(t)|0\rangle$

$|t|\sigma_x/\hbar \cdot t_{GATE} = \pi/2$

$\rightarrow |\psi(t_{GATE})\rangle = |1\rangle$

(3) MOVE ε FAST BACK TO ε_0

(4) READ OUT: QPC – IS THE CHARGE IN THE LEFT OR RIGHT DOT?

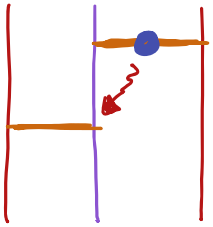
NOTE IF WE MOVE TO A FINITE $\varepsilon \neq 0$,

$$H = -|t|\sigma_x - \frac{1}{2}\varepsilon\alpha\sigma_z$$

↳ UNIVERSAL CONTROL OF THE QUBIT

HOW GOOD IS THIS QUBIT?

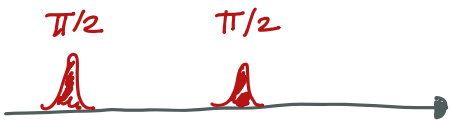
$|1\rangle$



DECAY OF THE EXCITED STATE

$T_1 \approx 10\,ns$

THE ENERGY GOES INTO PHONONS



$\frac{1}{2}(|0\rangle + |1\rangle) \rightarrow$ 50% $|0\rangle$ + 50% $|1\rangle$

$T_2 \approx 7\,ns$

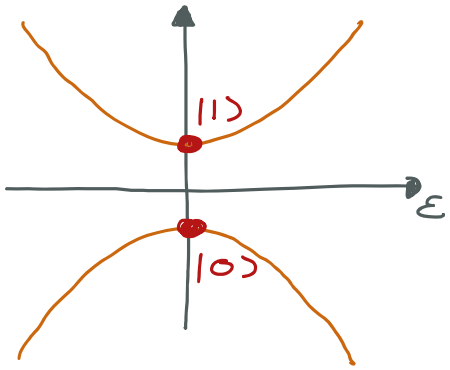
CHARGE NOISE CHANGES THE QUBIT FREQUENCY

$$H_0 = -\frac{1}{2}\varepsilon_0\alpha\sigma_z$$

↳ $\varepsilon_0 + \delta\varepsilon$

GOOD STRATEGY FIND AN INSENSITIVE SPOT

SECOND GENERATION CHARGE QUBIT



FIRST-ORDER INSENSITIVE TO NOISE IN $\varepsilon(t)$

MICROWAVE CONTROL:

$$H_0 = -|t|\sigma_x$$

$$H_D = \varepsilon(t)\,\sigma_z, \qquad \varepsilon(t) = A\cos(\omega_L t + \phi)$$

→ RABI HAMILTONIAN.

$$H = -\bar{\mu}\bar{B} \qquad \bar{B} = (B_x \cos\omega t, 0, B_z)$$

$$\bar{\mu} = -\frac{1}{2} g \mu_B \bar{\sigma}$$

$$H = -\frac{1}{2}\omega_0 \sigma_z + \Delta\sigma_x \cos(\omega t)$$

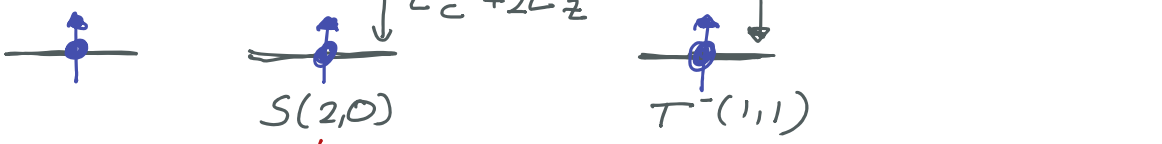
↳ RWA → $\tilde{H} = \frac{1}{2}(\omega_0 - \omega)\sigma_z + \frac{\Delta}{2}\sigma_x$

UNIVERSAL CONTROL

SPIN BLOCKADE

ENERGY LEVELS IN THE DOT IN THE PRESENCE OF MAGNETIC FIELDS

N = 1 N = 2

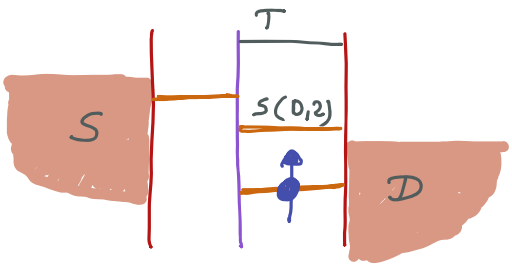


$S(2,0)$

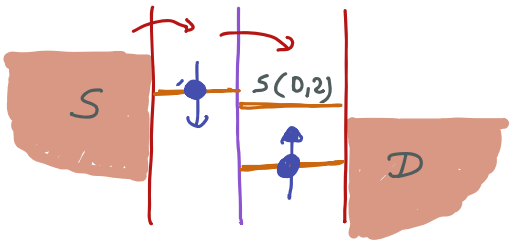
↳ THEY OCCUPY THE SAME ORBIT

$T^-(1,1)$

↳ THEY OCCUPY DIFFERENT ORBITS

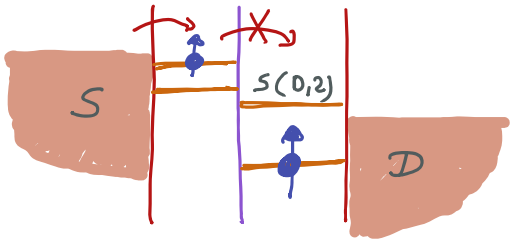


CASE 1:

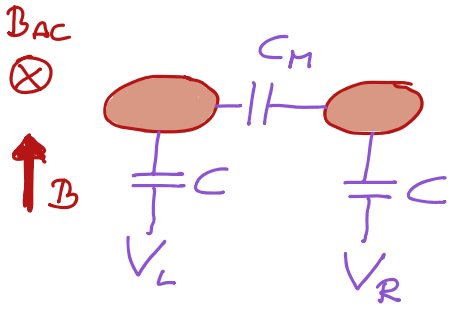


CURRENT FLOWS

CASE 2 :



CURRENT IS BLOCKED,

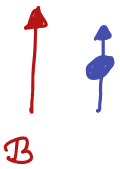


IF $\hbar\omega_L = \Delta E_Z$, THE CURRENT IS UNBLOCKED

ESR (ELECTRON SPIN RESONANCE)

SPIN QUBITS

THE SIMPLEST QUBIT:
SINGLE ELECTRON SPIN IN MAGNETIC FIELD



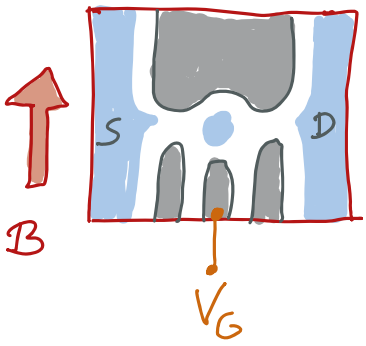
$$H = \frac{1}{2} g \mu_B \vec{\sigma} \cdot \vec{B} = \frac{1}{2} g \mu_B \sigma_z \cdot B_z$$

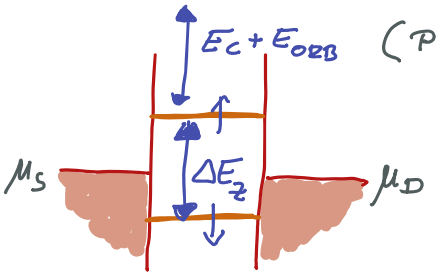
→ $|\uparrow\rangle$ & $|\downarrow\rangle$ ARE EIGENSTATES

QUBIT ENERGY $E_{01} = g \mu_B B_z = \Delta E_z$

$\sim 100\ \mu eV$ @ $B_z \sim 1T$

ZEEMAN ENERGY

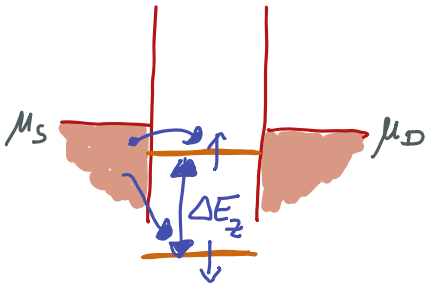




(PUTTING THE SECOND ELECTRON IN)

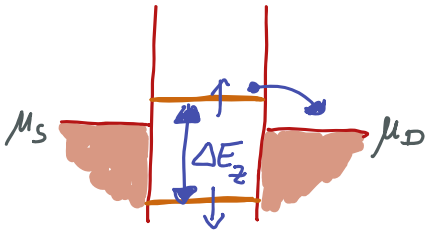
VERSION ONE (JUST TO MEASURE T_1, NO GATES)

① INITALIZE (UNCONTROLLED)

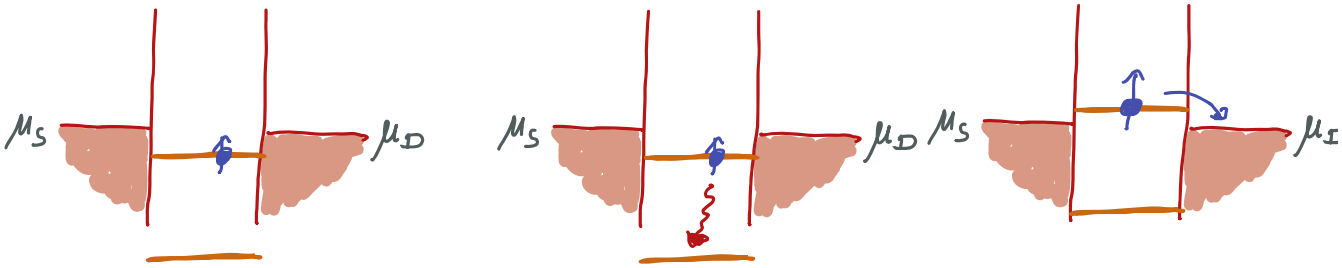


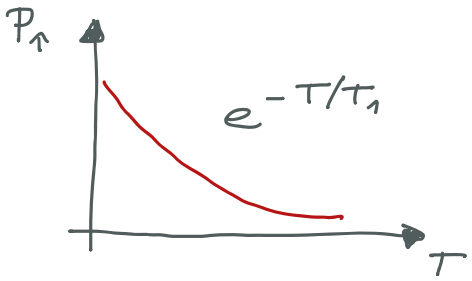
ELECTRON CAN HOP IN EITHER $|\uparrow\rangle$ OR $|\downarrow\rangle$ STATE

② READ OUT : CURRENT IF WE ARE IN THE $|\uparrow\rangle$ STATE (MEASURE IT WITH QPC)



③ MEASUREMENT :
- HOPE FOR $|\uparrow\rangle$ STATE
- WAIT TIME T
- MEASURE $|\uparrow\rangle$ STATE POPULATION





$T_1 \approx 100\,\mu s$

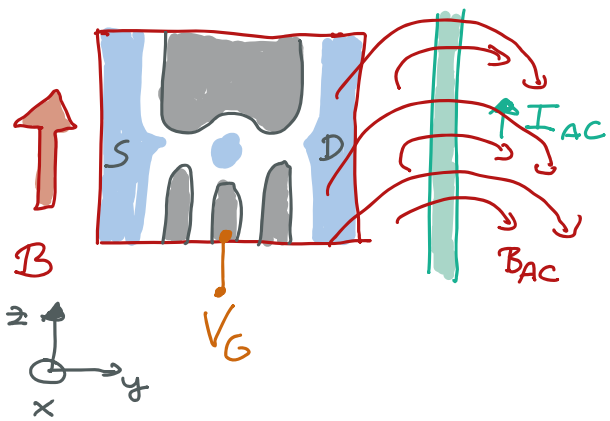
CHARGE QUBITS:
10 ns

4 ORDERS OF MAGNITUDE!

GaAs/AlGaAs (2004)

VERSION 2

MICROWAVE CONTROL

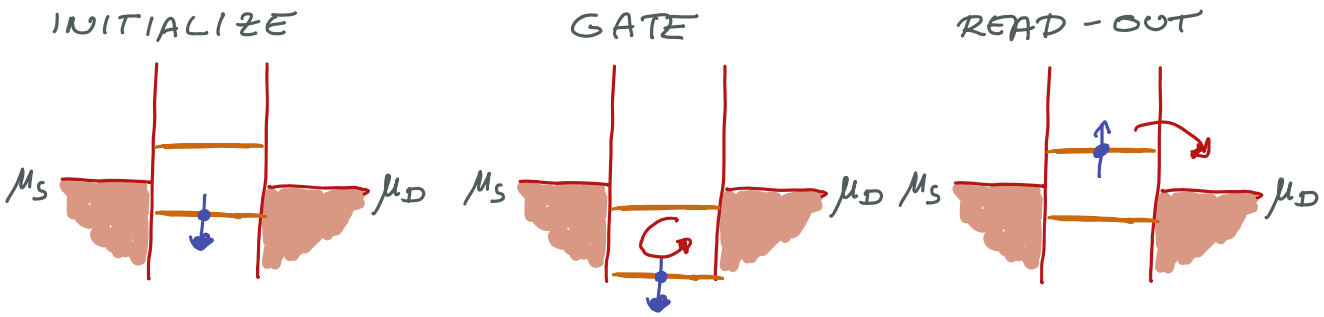


HIGH-FREQUENCY TRANSMISSION LINE

$$H = -\frac{1}{2} g \mu_B B \sigma_z +$$

$$-\frac{1}{2} g \mu_B B_{AC} \cos(\omega_c t) \sigma_x$$

RABI HAMILTONIAN.

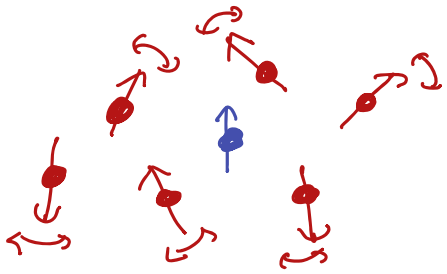


$T_{GATE} \approx 20\,ns$

$T_\varphi \approx 50 - 100\,ns$

↳ VERY SHORT COMPARED TO T_1

REASON: FLUCTUATING NUCLEAR SPIN FIELD IN GaAs



↓ SOLUTION: CHANGE THE HOST MATERIAL

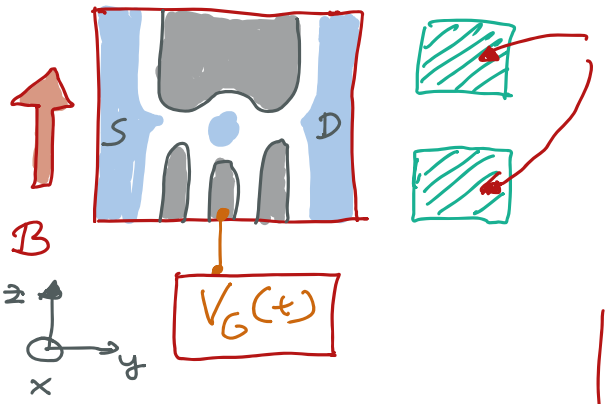
$^{28}Si/SiGe$ HETEROSTRUCTURE

↳ SMALL CONCENTRATION OF ^{29}Si (ISOTOPICALLY PURIFIED)

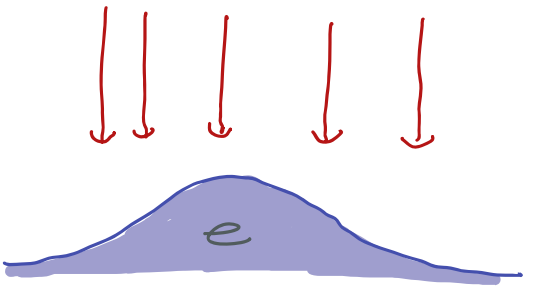
VERSION 3 (2018)

IMPROVEMENT : ELECTRICAL CONTROL

+ REDUCING NUCLEAR SPIN



MICROMAGNET:
CREATE INHOMOGENOUS
3D MAGNETIC FIELD



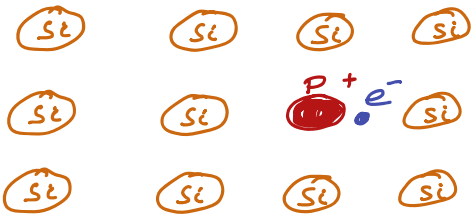
SHAKE THE ELECTRON

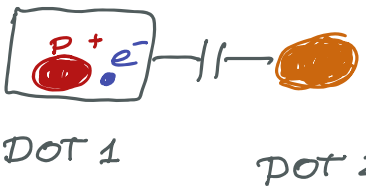
→ FEELS A CHANGING
MAGNETIC FIELD.

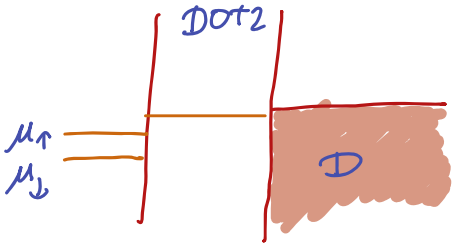
→ $T_\varphi = 20\mu s$

ALTERNATIVE ROUTE:

NATURAL QUANTUM DOT: DONOR







SAME STEPS AS BEFORE TO INIT, GATE, READ-OUT

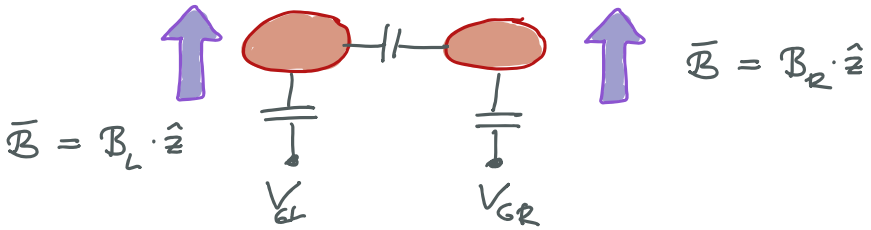
$T_\phi \approx 100\,ns$ BECAUSE OF THE NUCLEAR SPINS OF ^{29}Si

↓ PURIFYING

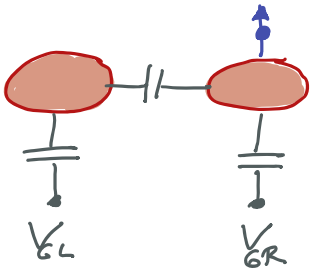
$T_\phi \approx 120\,\mu s$ (2014)

GATES ARE SLOW (5-10 μs)

TWO-QUBIT GATES WITH SPINS



ONE SPIN:



$$H = -\bar{\mu}\bar{B} = -\frac{1}{2}\underbrace{g\mu_B B_z}_{\hbar\omega_0}\cdot\sigma_z$$

GROUND STATE: $|\uparrow\rangle = |0\rangle$ $\quad E_0 = -\frac{1}{2}\hbar\omega_0$

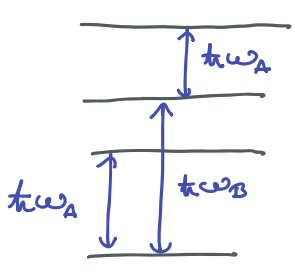
$|\downarrow\rangle = |1\rangle$ $\quad E_1 = \frac{1}{2}\hbar\omega_0$

TWO SPIN ⟷ TWO QUBITS

$|0\rangle_A = |\uparrow\rangle_A \quad E_0^A = -\frac{1}{2}\hbar\omega_A \qquad |0\rangle_B = |\uparrow\rangle_B \quad E_0^B = -\frac{1}{2}\hbar\omega_B$

$|1\rangle_A = |\downarrow\rangle_A \quad E_1^A = +\frac{1}{2}\hbar\omega_A \qquad |1\rangle_B = |\downarrow\rangle_B \quad E_1^B = \frac{1}{2}\hbar\omega_B$

ASSUME $\omega_A < \omega_B$, 4 STATES:



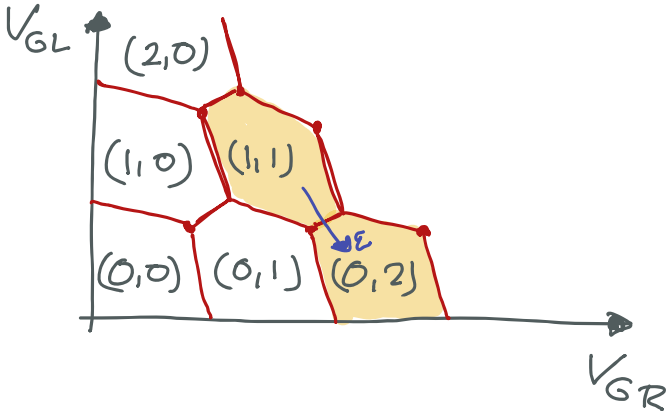
$|\downarrow\downarrow\rangle = |11\rangle$

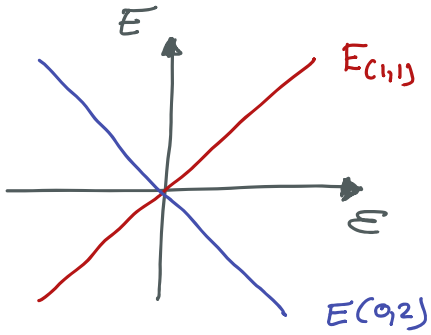
$|\uparrow\downarrow\rangle = |01\rangle$

$|\downarrow\uparrow\rangle = |10\rangle$

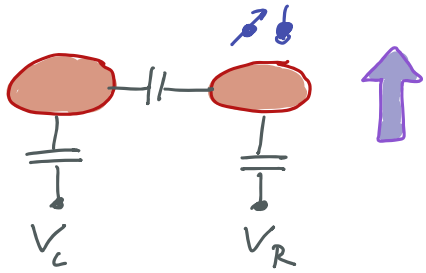
$|\uparrow\uparrow\rangle = |00\rangle$

- HOW TO IMPLEMENT THEM? DOUBLE DOTS
- HOW TO ENTANGLE THEM? CNOT, CZ
  iSWAP, $\sqrt{\text{SWAP}}$
  + SINGLE QUBIT GATES





TWO SPINS IN ONE DOT:



$\vec{B} = B_R \cdot \hat{z}$

GROUND STATE: SAME ORBITAL + OPPOSITE SPIN

$$S(0,2) = |1s\rangle|1s\rangle \cdot \left[|\bullet, \uparrow\downarrow\rangle - |\bullet, \downarrow\uparrow\rangle \right]$$

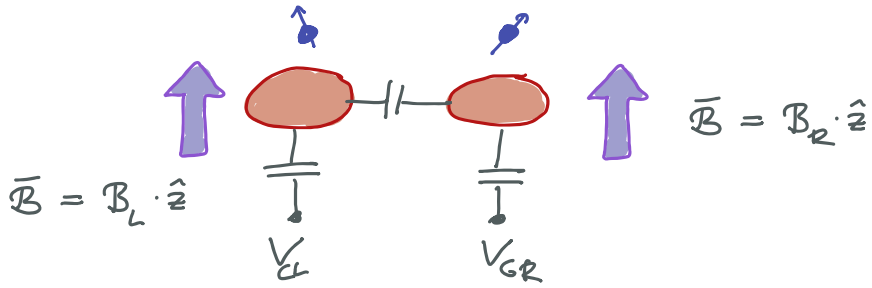
L R L R

EXCITED STATE:

$$\left[|1s\rangle|2s\rangle - |2s\rangle|1s\rangle \right] \cdot \begin{cases} |\bullet, \uparrow\uparrow\rangle \\ |\bullet, \uparrow\downarrow\rangle + |\bullet, \downarrow\uparrow\rangle \\ |\bullet, \downarrow\downarrow\rangle \end{cases}$$

THE ENERGY OF THESE STATES ARE MUCH HIGHER $+ E_{ORB}(2s)$

TWO SPINS IN TWO DOTS



(1,1) STATE:

BOTH ARE IN THE GROUND STATE OF THE DOT:

$$S(1,1) = \left[|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle\right] \quad \text{SINGLET}$$

$$\left.\begin{aligned} T^0(1,1) &= \left[|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle\right] \\ T^+(1,1) &= |\uparrow,\uparrow\rangle \\ T^-(1,1) &= |\downarrow,\downarrow\rangle \end{aligned}\right\} \quad \text{TRIPLET}$$

IF $\bar{B} = 0$ DEGENERATE $E_{(1,1)}$

IF $\bar{B} \neq 0$ $[B_L = B_R]$

$$\boxed{\begin{aligned} S(1,1) &\Rightarrow 0 \\ T^0(1,1) &\Rightarrow 0 \end{aligned}}$$

$$T^+(1,1) \Rightarrow -\tfrac{1}{2} g\mu_B (B_L + B_R)$$

$$T^-(1,1) \Rightarrow +\tfrac{1}{2} g\mu_B (B_L + B_R)$$

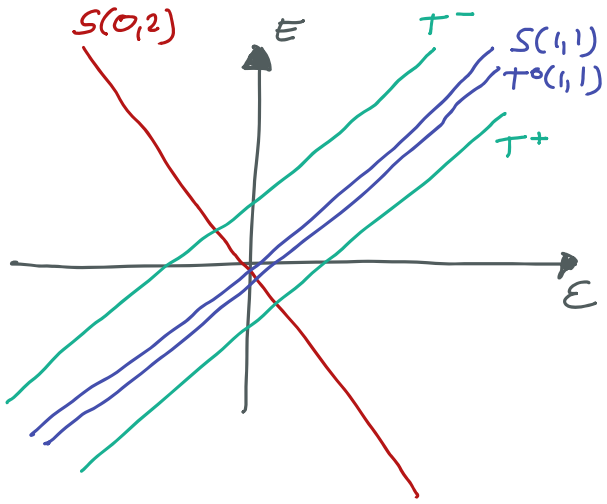
SINGLET-TRIPLET ENCODING

$S(1,1) \triangleq |0\rangle$

$T(1,1) = |1\rangle$

IF $B_L \neq B_R$: ΔB

$$-\tfrac{1}{2} g\mu_B \left[B_L - B_R - (-B_L + B_R)\right] =$$



CHARGE NOISE ?

KINETIC EXCHANGE

(1) $|\uparrow,\downarrow\rangle \longleftrightarrow |\cdot,\uparrow\downarrow\rangle$

$$\langle\uparrow,\downarrow|\,\hat{H}_{\text{JUMP}}\,|\cdot,\uparrow\downarrow\rangle = t$$

(2) $|\downarrow,\uparrow\rangle \longleftrightarrow |\cdot,\uparrow\downarrow\rangle$

$$|\downarrow,\uparrow\rangle = -|\uparrow,\downarrow\rangle$$

$$\langle\downarrow,\uparrow|\,\hat{H}_{\text{JUMP}}\,|\cdot,\uparrow\downarrow\rangle = -t$$

(3) $|\uparrow,\uparrow\rangle \longleftrightarrow |\cdot,\uparrow\downarrow\rangle$

$$\langle\uparrow,\uparrow|\,\hat{\mu}_{JUMP}\,|\cdot,\uparrow\downarrow\rangle = 0$$

(4) $|\downarrow,\downarrow\rangle \longleftrightarrow |\cdot,\uparrow\downarrow\rangle$

$$\langle\downarrow,\downarrow|\,\hat{\mu}_{JUMP}\,|\cdot,\uparrow\downarrow\rangle = 0$$

SINGLET - SINGLET INTERACTION:

$$\langle S(1,1)|\,\hat{\mu}_{JUMP}\,|S(0,2)\rangle =$$

$$\left[\langle\uparrow\downarrow| - \langle\downarrow\uparrow|\right]\hat{\mu}_{JUMP}\left[|\cdot,\uparrow\downarrow\rangle - |\cdot,\downarrow\uparrow\rangle\right] =$$

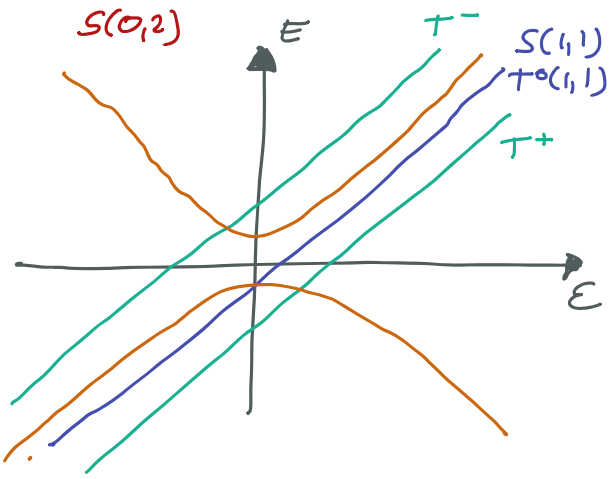
$$= t + t + t + t = 4t$$

SINGLET - TRIPLET INTERACTION

$$\langle T(1,1)|\,\hat{\mu}_{JUMP}\,|S(0,2)\rangle =$$

$$\left[\langle\uparrow\downarrow| + \langle\downarrow\uparrow|\right]\hat{\mu}_{JUMP}\left[|\cdot,\uparrow\downarrow\rangle - |\cdot,\downarrow\uparrow\rangle\right] =$$

$$= t - t - t + t = 0$$



SIDE NOTE: SINGLET-TRIPLET QUBITS

→ PROTECTED AGAINST GLOBAL FLUX NOISE.

IN THE TWO-QUBIT BASIS:

$T^- = |\downarrow\downarrow\rangle$

$S(1,1) = (|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle)$

$T(1,1) = (|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)$

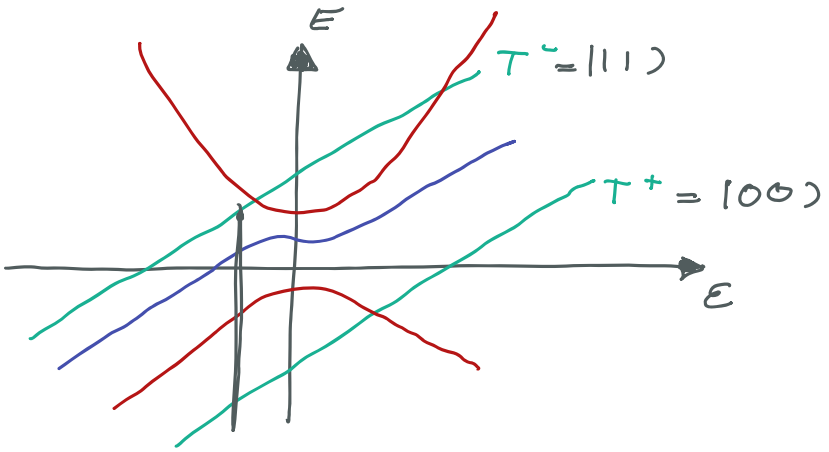
$T^+ = |\uparrow\uparrow\rangle$

$|11\rangle = |\downarrow\downarrow\rangle$

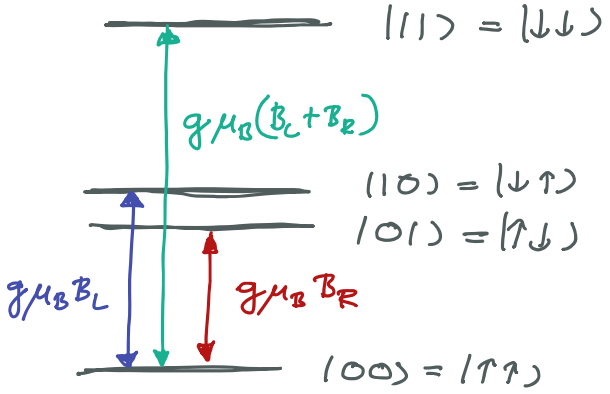
$|10\rangle = |\downarrow\uparrow\rangle$

$|01\rangle = |\uparrow\downarrow\rangle$

$|00\rangle = |\uparrow\uparrow\rangle$



$t = 0$ (ISOLATED DOTS)

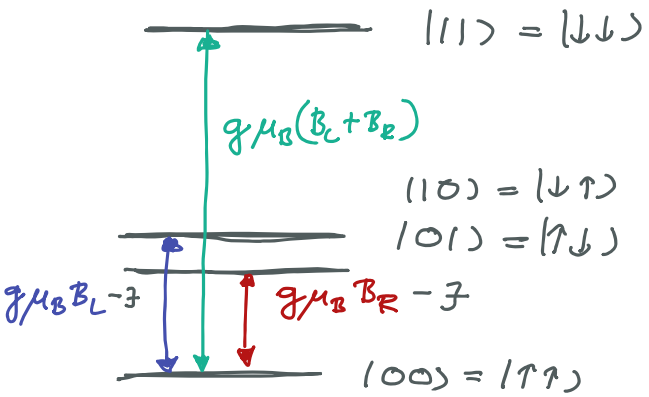


$$E(|10\rangle - |00\rangle) = E(|11\rangle - |01\rangle) = g\mu_B B_L$$

→ SINGLE QUBIT GATE $B_{Lx} = \Omega \cos(\omega_L t)$

$\hbar\omega_L = g\mu_B B_L$ DOES NOT CARE ABOUT THE STATE OF

$t \neq 0$



$$E(|01\rangle - |00\rangle) \neq E(|11\rangle - |10\rangle)$$

EXCITING AT $\omega_L = (g\mu_B B_R + J)/\hbar$
WORKS AS A SINGLE QUBIT GATE
ONLY IF THE LEFT SPIN IS EXCITED

$$\text{CNOT} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 0 & 1 \\ & & 1 & 0 \end{bmatrix}$$

TWO INTERACTING SPINS

REDUCING THE GLOBAL FLUX NOISE

→ SINGLET - TRIPLET QUBITS

ELECTRONS ARE FERMIONS:

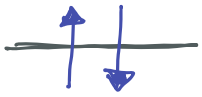
$$\Psi(1,2) = -\Psi(2,1)$$

SPACE PART + SPIN PART

$$\Psi(\vec{r}, \sigma) = \Psi_{SPACE}(\vec{r}) \cdot \Psi_{SPIN}(\sigma)$$

EXAMPLE: HELIUM ATOM

GROUND STATE: $1s^2$



$$\Psi(1,2) = \underbrace{\Psi_{1s}(r_1)\,\Psi_{1s}(r_2)}_{\text{SAME ORBITAL}} \times \underbrace{\left(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right)}_{\text{SINGLET STATE}}$$

INDEED $\Psi(1,2) = -\Psi(2,1)$

EXCITED STATE: $1s2s$

SINGLET

- SYMMETRIC SPATIAL × ANTISYMM SPIN

$$\underbrace{\left[\psi_{1s}(r_1)\psi_{2s}(r_2) + \psi_{2s}(r_1)\psi_{1s}(r_2)\right]}_{\psi_S} \times \left(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2\right)$$

TRIPLET

- ANTISYMMETRIC SPATIAL × SYMMETRIC SPIN

$$\underbrace{\left[\psi_{1s}(r_1)\psi_{2s}(r_2) - \psi_{2s}(r_1)\psi_{1s}(r_2)\right]}_{\psi_T} \times \left(|\uparrow\rangle_1 |\uparrow\rangle_2\right) \times \left(|\downarrow\rangle_1 |\downarrow\rangle_2\right) \times \left(|\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2\right)$$

ENERGY OF THESE STATES

ELECTRON - ELECTRON REPULSION

$$U = \frac{e^2}{|r_1 - r_2|}$$

WHEN $r_1 = r_2$ $\quad \psi_T = 0$, $\quad \psi_S \neq 0$

→ SINGLET STATE HAS HIGHER ENERGY. (HUND'S RULE)

THE ENERGY DIFFERENCE IS THE DIRECT EXCHANGE ENERGY.